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Undistorted Configurations for Two Classes of Crystals

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Abstract

In this paper we describe the concept of material objectivity, material symmetry and undistorted configurations using the work "Non-linear elastic deformations" of R. W. Ogden .Using the relations between two undistorted configurations we find what kind of deformations keep the symmetry group for triclinic and monoclinic crystals-two classes of anisotropic solids.

Key words: *undistorted configurations,symmetry group, isotropic material, anisotropic material*

Introduction

This paper is based on a problem proposed in [3] which intends to find the undistorted configurations for solids crystal class with the group of material symmetry triclinic or monoclinic.

All the introduction follows the presentation of R.W.Ogden [2].

We use the Cauchy elastic material who is an elastic simple material.

Definition 1. An elastic simple material is one at each material point of which the state of stress in the current configuration is determined solely by the state of deformation of this configuration relative to an arbitrary choice of reference configuration.

Definition 2. The Cauchy elastic material is caracterized by the property: the stress is not derivable from a scalar potential function.

As a result of this definition the constitutive law for a Cauchy elastic material is:

$$T\{\chi(X,t),t\} = G(F;X)\,, \qquad (1)$$

where

$$F(X,t) = Grad\chi(X,t)\,. \qquad (2)$$

X is a material point fixed in the reference configuration and t is the time.

We refer to G as the response function of the Cauchy elastic material relative to the chosen reference configuration , k and to simplify the notation we rewrite (1) as

$$T = G(F)\,. \qquad (3)$$

Relation (3) shows that the mechanical properties of the material are characterized by the function G who is a mapping from the space of invertible second-order tensors to the space of symmetric second order tensors.

For an observer transformation described by Q (tensor orthogonal) the deformation gradient transforms according to relation:

$$F^* = QF \tag{4}$$

and the Cauchy stress tensor :

$$T^* = QTQ^T . \tag{5}$$

Material objectivity states it that the material properties are independent of observer and this means that the function G is the same for all observers and hence

$$T^* = G\ (F^*) . \tag{6}$$

It follows that

$$G(QF) = QG(F)Q^T \tag{7}$$

for all proper orthogonal Q and arbitrary deformation gradients F.

Let it be

$$F = RU \tag{8}$$

the polar decomposition of the deformation gradient with R a proper orthogonal tensor and U a symmetric positive-definite tensor. It follows from (2) and (7) that

$$T = G(F) = RG(U)R^T . \tag{9}$$

The relation (9) show that G is fully determined by its restriction to a domain consisting of positive –definite symmetric second-order tensors which states that intrinsic properties of a Cauchy elastic material are not influenced by the rotational part R of the deformation.

Material Symmetry and Undistorted Configurations

Let it be F and F' the deformation gradients relative to the reference configuration k si $\bar{k}$, with P_0 denoting the deformation gradient from k to $\bar{k}$. It follows that:

$$G'(FP_0^{-1}) = G(F) \tag{10}$$

for all deformation gradients F and each given P_0 .

Suppose that for some given P_0 the response of the material relative to $\bar{k}$ and k is the same concerning any mechanical test.This means that $G' \equiv G$ and (10) becomes

$$G(FP_0^{-1}) = G(F) \tag{11}$$

for all deformation gradients F and the P_0 in question.

Consider the set Ψ of invertible second-order tensors K for which

$$G(FK) = G(F) \tag{12}$$

for all deformations gradients F .

Proposition 1. Ψ has the structure of a group.

Demonstration. If $K,\overline{K} \in \Psi$ then

$$G(FK\overline{K}) = G(FK) = G(F) \tag{13}$$

and hence $K\overline{K} \in \Psi$.

Let $K \in \Psi$; it result that K^{-1} exists and

$$G(F) = G(FKK^{-1}) = G(FK^{-1}K) = G(FK^{-1}) \tag{14}$$

and hence $K^{-1} \in \Psi$ .The identity $I \in \Psi$ since *FI=F.*

Thus, the set Ψ forms a group and it is called the symmetry group of the material relative to the reference configuration k .

In [2] Ogden show that the effect on Ψ of a change of reference configuration from k to $\bar{k}$, with $F' = FP_0^{-1}$ is:

$$\overline{\Psi} = P_0 \Psi P_0^{-1}, \tag{15}$$

where Ψ' is the symmetry group of the material relative to k .He considers that the symmetry group of a Cauchy elastic material is unaffected by a change of reference configuration corresponding to an arbitrary pure dilatation $P_0 = p_0 I$.

At this point we emphasize that G is defined over the set of invertible second-order tensors with positive determinant . This restriction is consistent with the requirement that we ruled out from consideration deformations which change the relative orientations of triads of line elements.

The proper orthogonal group is

$$Ort^+ = \{A \in Lin \big| AA^T = I, \det(A) = 1\}. \tag{16}$$

Definition 3. An isotropic elastic material is an elastic material whose symmetry group contains the proper orthogonal group for at least one reference configurations k

$$\Psi \supset Ort^+. \tag{17}$$

Such a reference configurations is called undistorted configurations ; the mechanical response of the material exhibits no preferred direction.

If the symmetry group Ψ is equal to the proper orthogonal group ,the material is said to be an isotropic elastic solid.

For a number of solid materials there exist reference configurations relative to which the symmetry groups of their response functions contain strict subgroups of the proper orthogonal group .

Definition 4. The anisotropic elastic material is an elastic material for which there is a reference configuration k with the symmetry group Ψ that verifies:

$$\Psi \subset Ort^+; \Psi \neq Ort^+. \tag{18}$$

Reference configurations with the above property are called undistorted configuration .

The anisotropic elastic material has one or more preferred directions.

The anisotropic elastic solid which possess three preferred directions in an undistorted configurations is called crystal classes.

Proposition 2. Let k be a undistorted configuration for a anisotropic elastic solid with $\Psi \subset Ort^{+}, \Psi \neq Ort^{+}$.The set of undistorted configurations $\bar{k}$ with $\Psi = \overline{\Psi}$ is characterized by deformations λ with $P = \nabla\lambda(X)$ having the form $P = R_0 U_0$ and

$$U_0 Q_0 = Q_0 U_0, \tag{19}$$

$$R_0 \Psi R_o^{\ T} = \Psi \tag{20}$$

for all $Q_0 \in \Psi$.

The demonstration is finding in [2].

Undistorted Configurations for Crystals with Triclinic System

Let be a orthonormate base $\{i_1, i_2, i_3\}$.

The class of crystals with triclinic system is described by:

$$\Psi = \{I, -I\} . \tag{21}$$

The aim of this paper is to find all the undistorted configurations for such material using the proposition 2.

For Ψ in (21) it result that (19) holds for all U_0 positive-definite tensor and (20) holds for all R_0 in Ort^{+}.

Conclusion. If a solid material has a configuration k with Ψ corresponding to a crystal triclinic then for all the configurations the material is a crystal triclinic.

Undistorted Configurations for Crystals with Monoclinic System

The class of crystals with monoclinic system is described by :

$$\Psi = \{-I, +I, R_k^{\pi}, -R_k^{\pi}\} . \tag{22}$$

The rotation R_k^{ρ} is described in the base $\{i_1, i_2, i_3\}$ by matrix:

$$\begin{pmatrix} \cos\rho & \sin\rho & 0 \\ -\sin\rho & \cos\rho & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{23}$$

$$R_k^{\pi} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad -R_k^{\pi} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \tag{24}$$

Using the relations:

$$U_0 R_k^{\pi} = R_k^{\pi} U_0$$
$$U_0(-R_k^{\pi}) = (-R_k^{\pi})U_0 , \tag{25}$$

it results

$$U_{13} = U_{23} = 0 . \tag{26}$$

The relation (26) show that U_0 have the matrix associated in the base $\{i_1, i_2, i_3\}$:

$$\begin{pmatrix} U_{11} & U_{12} & 0 \\ U_{12} & U_{22} & 0 \\ 0 & 0 & U_{33} \end{pmatrix} . \tag{27}$$

The restriction for the rotation R_0 is obtained using the relation (20):

$$\begin{cases} R_0 I R_0^T = I \\ R_0 R_k^{\pi} R_0^T = R_k^{\pi} \end{cases} or \begin{cases} R_0 I R_0^T = I \\ R_0 R_0^{\pi} R_0^T = -R_k^{\pi} \end{cases} . \tag{28}$$

Restrictions (28) is equivalent to

$$R_0 R_k^{\pi} = R_k^{\pi} R_0 \tag{29}$$

or

$$R_0 R_k^{\pi} = -R_k^{\pi} R_0 . \tag{30}$$

Using (24) in (29) on obtained

$$R_0 = \begin{pmatrix} R_{11} & R_{12} & 0 \\ R_{21} & R_{22} & 0 \\ 0 & 0 & R_{33} \end{pmatrix} , \tag{31}$$

with

$$\begin{cases} R_{11}^2 + R_{12}^2 = 1 \\ R_{11}R_{21} + R_{22}R_{12} = 0 \\ R_{21}^2 + R_{22}^2 = 1 \\ R_{33} = \pm 1 \end{cases} . \tag{32}$$

The form (31) with the restrictions (32) show that R_0 or - R_0 is a rotations around the axis i_3.

Using (24) in (30) we obtain $\det(R_0) = 0$ incompatible with a rotation.

Conclusion. If a solid material has a configuration k with Ψ corresponding for a system monoclinic then another undistorted configuration $\bar{k}$ is related by k with U_o on the form

$$\begin{pmatrix} U_{11} & U_{12} & 0 \\ U_{12} & U_{22} & 0 \\ 0 & 0 & U_{33} \end{pmatrix} \tag{33}$$

and R_0 or - R_0 a rotations around the axis i_3.

U_0 in the form (33) is a deformation who transform the material elements localized in the plain (i_1, i_2) in material element localized in same plain.If $U_{33} = 1$ then U_o is a plain deformation.

This conclusions show what kind of deformations keep the properties of the monoclinic and triclinic crystals.

References

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Configuraţii Nedistorsionate pentru Două Clase de Cristale

Rezumat

În acest articol sunt descrise conceptele de obiectivitate materială, simetrie materială şi configuraţii nedistorsionate pornind de la lucrarea „Non-linear elastic deformations”, autor R..W. Ogden. Folosind relaţiile dintre două configuraţii nedistorsionate am determinat tipurile de deformaţii care păstrează grupul de simetrie pentru cristale triclinice şi monoclinice - două clase de solide anizotrope.