BULETINUL Universităţii Petrol – Gaze din Ploieşti	Vol. LX No. 1/2008	73 - 76	Seria Matematică - Informatică - Fizică

Study of the Flow Over an Oscillating Disk

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Abstract

This paper aims at obtaining exact analytical solutions for the flow over an oscillating disk in the presence of partial slip and a porous medium. Some flow generated by certain special oscillation is also included in each case. The research have found that the velocity profile is of the wave nature and that amplitude of the wave decreases when the partial slip parameter increases.

Key words: *flow, oscillation, velocity profile*

Introduction

We consider axis y to be natural to consider the system of cartesian coordinate so that the axis x to be parallel to the rigid plan and the it. The fluid of 2nd degree occupies the porous space (y>0). At the same time, we consider the unidirectional flow between two periodic oscillations along to an infinite disk.

Theoretic Details

In this case we don't have flow on the directions y and z and the speed will be written:

$$V = (u(y,t),\ 0,\ 0)\,, \tag{1}$$

where u is the component on the speed axis x. The continuity equation (1) is perfectly accomplished, and the impulse equation [1] in the absence of pressure gradient is:

$$\rho\frac{\partial u}{\partial t} = \mu\frac{\partial^2 u}{\partial y^2} + \rho d^2\frac{\partial^3 u}{\partial t\partial y^2} - \frac{\Phi}{k}\left(\mu + \rho d^2\frac{\partial}{\partial t}\right)u\,, \tag{2}$$

where K is the permeability, Φ - porosity, 0<Φ<1.

In the above equation the 3rd term of the right side of equal represents the viscous amortization which is produced between the microstructures components of the porous medium. As a matter of fact this is a measure of the resistance to flow. I have considered as well: $\alpha_1 = \rho d^2$, where ρ is the fluid density and d is the elasticity coefficient. If we consider the limit conditions:

$$u(y,t) - \Omega\left(\frac{\partial u}{\partial y} + \frac{\rho d^2}{\mu}\frac{\partial^2 u}{\partial t\partial y}\right) = u_0 f(t) \text{ for } y = 0, \tag{3}$$

$$u(y,t) \to 0 \quad \text{if } y \to \infty, \tag{4}$$

where Ω is the drift coefficient; u_0 – reference speed, and $f(t)$ describes the periodic oscillations.

The Fourier series for $f(t)$ is:

$$f(t) = \sum_{k=-\infty}^{\infty} a_k e^{ikq_0 t}, \tag{5}$$

where the coefficient a_k is deduced from the relation:

$$a_k = \frac{1}{T_0} \int_{T_0} f(t) e^{ikq_0 t} \mathrm{d}t, \tag{6}$$

with the fundamental frequency:

$$q_0 = \frac{2\pi}{T_0}, \text{ where } T_0 \text{ represents the period } f(t).$$

From a nondimensional point of view, the equations (2), (3), (4) must comply with:

$$\bar{u} = \frac{u}{u_0} \quad \bar{y} = \frac{yu_0}{\nu} \quad \bar{t} = \frac{tu_0^2}{\nu}, \tag{7}$$

where ν is the kinematic viscosity.

If we propose to express the equations in nondimensional form, these will be written:

$$(1+\alpha)\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + l^2 \frac{\partial^3 u}{\partial y^2 \partial t} - \beta^2 u, \tag{8}$$

$$u - \gamma\left(\frac{\partial u}{\partial y} + l^2 \frac{\partial^2 u}{\partial y \partial t}\right) = \sum_{k=-\infty}^{\infty} a_k e^{ik\omega_0 t}, \text{ when } \; y = 0, \tag{9}$$

$$u \to 0 \text{ when } y \to \infty. \tag{10}$$

In this equation the limits were omitted for simplicity, and it has been considered that:

$$\alpha = \frac{\Phi d^2}{K}; \; l^2 = \frac{du_0^2}{\nu^2}; \; \beta^2 = \frac{\nu^2 \Phi}{u_0^2 K}; \; \gamma = \frac{\Omega u_0}{\nu}; \; \omega_0 = \frac{\nu q_0}{u_0^2}. \tag{11}$$

We will solve the problem with the help of Fourier transforms.

The Fourier temporal transform is defined as:

$$\Psi(y,\omega) = \int_{-\infty}^{\infty} u(y,t) e^{-i\omega t} \mathrm{d}t, \tag{12}$$

$$u(y,t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Psi(y,\omega) e^{i\omega t} \mathrm{d}\omega, \tag{13}$$

where ω is the frequency. Using the equation (12) the equations (8)-(10) become:

$$\frac{\mathrm{d}^2\Psi}{\mathrm{d}y^2} - \eta^2 \omega = 0 \tag{14}$$

$$\Psi - \gamma(1 + i\omega l^2)\frac{\mathrm{d}\Psi}{\mathrm{d}y} = 2\pi \sum_{k=-\infty}^{\infty} a_k \delta(\omega - k\omega_0) \text{ for } y=0 \tag{15}$$

$$\Psi \to 0 \text{ if } y \to \infty, \tag{16}$$

where:

$$\eta^2 = \frac{\lfloor(1+\alpha)\omega - i\beta^2\rfloor(i+\omega l^2)}{1+(\omega l^2)^2}, \tag{17}$$

and δ is the Dirac function.

In order to satisfy the equations (15) and (16), the solutions of the (14) equation must be:

$$\Psi = 2\pi \sum_{k=-\infty}^{\infty} a_k \frac{\delta(\omega-\omega_0)e^{-\eta y}}{1+\gamma(1+i\omega l^2)\eta}. \tag{18}$$

By replacing the expression in the equation (13) and using the δ function properties, we obtain:

$$u = \sum_{k=-\infty}^{\infty} a_k \frac{\exp[-m_k y + i(k\omega_0 t - n_k y)]}{[1+\gamma(1+ik\omega l^2)\eta_k]}, \tag{19}$$

where m_k and n_k are the real part, and respectively virtual part of η_k offered by:

$$m_k^2 = \frac{1}{2[1+(k\omega_0 l^2)^2]}\sqrt{[(1+\alpha)k^2\omega_0^2 l^2+\beta^2]^2+[(1+\alpha)k\omega_0-\beta^2 l^2 k\omega_0]^2} + \tag{20}$$

$$+(1+\alpha)k^2\omega_0^2 l^2+\beta^2,$$

$$n_k^2 = \frac{1}{2[1+(k\omega_0 l^2)^2]}\sqrt{[(1+\alpha)k^2\omega_0^2 l^2+\beta^2]^2+[(1+\alpha)k\omega_0-\beta^2 l^2 k\omega_0]^2} - \tag{21}$$

$$-(1+\alpha)k^2\omega_0^2 l^2+\beta^2,$$

$$\eta_k = \eta|_{\omega=k\omega_0} \text{ and } 1+k\omega_0 l^2 \neq 0.$$

The flow area along with the disk oscillating disk might be described with the help of equation (16) by extraction of Fourier transforms (converters).

We consider the following five oscillation periods:

1. $f(t) = e^{i\omega_0 t}$, $a_1 = 1$ and $a_k = 0$ $(k \neq 1)$,
2. $f(t) = \cos\omega_0 t$, $a_1 = a_{-1} = \frac{1}{2}$ and $a_k = 0$,
3. $f(t) = \sin\omega_0 t$, $a_1 = a_{-1} = \frac{1}{2i}$ and $a_k = 0$,
4. $f(t) = \begin{cases} 1, & |t| < T_1 \\ 0, & T_1 < |t| < \frac{T_0}{2} \end{cases}$, $a_0 = \frac{2\pi}{T_0}$ and $a_k = \frac{\sin(k\omega_0 T_1)}{k\pi}$,
5. $f(t) = \sum_{k=-\infty}^{\infty}\delta(t-kT_0)$, $a_k = \frac{1}{T_0}$ for all k.

With the help of equation (19) the speed component can be expressed for all cases:

$$u_1 = \frac{\exp[-m_1 y + i(k\omega_0 t - n_1 y)]}{1+\gamma(1+i\omega_0 l^2)\eta_1}, \tag{22}$$

$$u_2 = \frac{1}{2}\left[\frac{\exp[-m_1 y + i(\omega_0 t - n_1 y)]}{1+\gamma(1+i\omega_0 l^2)\eta_1} + \frac{\exp[-m_{-1} y - i(\omega_0 t + n_{-1} y)]}{1+\gamma(1-i\omega_0 l^2)\eta_{-1}}\right], \tag{23}$$

$$u_3 = -\frac{i}{2}\left[\frac{\exp[-m_1 y + i(\omega_0 t - n_1 y)]}{1+\gamma(1+i\omega_0 l^2)\eta_1} - \frac{\exp[-m_{-1} y - i(\omega_0 t + n_{-1} y)]}{1+\gamma(1-i\omega_0 l^2)\eta_{-1}}\right], \qquad (24)$$

$$u_4 = \frac{1}{\pi}\sum_{k=-\infty}^{\infty}\frac{\sin(k\omega_0 T_1)}{k}\cdot\frac{\exp[-m_k y + i(k\omega_0 t - n_k y)]}{1+\gamma(1+i\omega_0 l^2)\eta_k},\ k\neq 0, \qquad (25)$$

$$u_5 = \frac{1}{T_0}\sum_{k=-\infty}^{\infty}\frac{\exp[-m_k y + i(k\omega_0 t - n_k y)]}{1+\gamma(1+i\omega_0 l^2)\eta_k}. \qquad (26)$$

Conclusions

We have presented an incompressible second grade fluid using modified Darcy's law. A general periodic oscillation is employed to induce the motion. The exact solutions of the problems have been obtained using the Fourier transform.

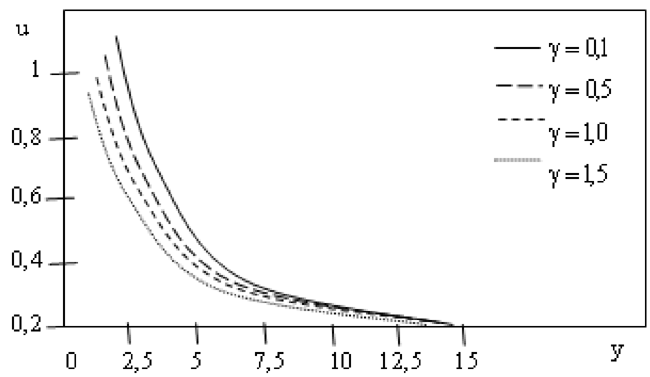

Fig. 1. Variation γ on the velocity profile, $\alpha = 0{,}1$, $\omega_0 = 0{,}5$, $\beta = 0{,}5$, $l = 1, t = 0{,}5$

The wave is rapidly damped in the interior of the fluid (fig.1), that is, the amplitude decreases exponentially with y.

References

1. Ionescu, D., Isbăşoiu, E., Ioniţă, I. - *Mecanica fluidelor şi maşini hidraulice*, Editura Didactică şi Pedagogică, Bucureşti, 1980
2. Landau, L., Lifchitz, E. - *Physique teoretique – Mecanique*, Editions de Moscov, 1969

Studiul curgerii deasupra unui disc care oscilează

Rezumat

Ne propunem să determinăm soluţiile analitice exacte pentru curgerea periodică generalizată a unui fluid de gradul II în condiţia unei alunecări parţiale şi a unui mediu poros. Unele curgeri generează oscilaţii speciale care sunt luate în considerare în acest caz. Este găsit profilul vitezelor pentru care amplitudinea mişcării ondulatorii scade cu creşterea parametrului de alunecare parţială.