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Interne Approximation for a Variational Inequality Problem

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Abstract

In this paper we describe a modality to transform a variational inequality in infinite dimension into a problem in finite dimension. We use this theoretical concept for a concrete problem of elasto-plasticity for a simple domain. We use a Math-lab version to programming the calculus of triples integrals witch appear in finite element procedures.

Key words: *variational inequality, finite element method, isotropic material.*

Introduction

The mathematical models of science and engineering mainly take the form of differential equations with constraints. Usually this kind of problems is transformed in variational inequality problem. To use mathematical models on a computer we need to develop numerical methods. In this article we present a modality to transform a variational inequality in infinite dimension into a problem in finite dimension using the finite element method (FEM).

The FEM is used by many authors for finding numerical solution of partial differential equations with boundary conditions transformed into variational problem. We obtained variational inequality when the partial differential equation appears with inequality constraints. We don't discuss in this article about the conditions of existence and uniqueness of solutions. We find in [4] and [6] a general discussion about the existence and uniqueness for variational problem and optimization algorithms. Such problems are solved numerically using the FEM by many authors like *Cocu M.,Pratt E , Simo J.C , Glowinsky R Lions,J.L. Tremolieres R* **.**

In [2] the author *Sanda Cleja Tigoiu* obtain the variational inequalities for an elasto-plastic model characterized by an initial anisotropy using the constitutive framework of materials with relaxed configurations and internal variables. Another author who transforms elasto-plastic problems into variational inequalities is J.C.Simo.In [8] he develops numerical models for classical plasticity and for multiplicative plasticity.

In this article we use a simpler constitutive model with infinitesimal deformation and isotropic hardening. The material solid is isotropic (the comportment is the same in all directions) and we use the Mises yield condition who is unaffected by a superposition of an arbitrary hydrostatic pressure. The variational inequality who describes the comportment of the body at a moment of time fixed is writing without the demonstration. The goal is to obtain the problem in finite dimension using the shape functions associated with a simple triangulation.

Interne Approximation

Let V be a Hilbert space, $(\cdot,\cdot)$ the inner scalar on $V \times V$ and $\|\cdot\|$ the norm induced.

Definition 1. $a : V \times V \to R$ is coercive if exists $\alpha > 0$, $a(v,v) \geq \alpha \|v\|^2 \forall v \in V$.

Let be $a : V \times V \to R$ a bi-linear, continuous, symmetric and coercive form and $f : V \to R$ linear application .We introduce the functional:

$$J : V \to R, J(v) = \frac{1}{2} a(v,v) - f(v) . \tag{1}$$

Let be K a convex and closed set, $K \subset V$.

Proposition 1. If $a : V \times V \to R$ is a bi-linear, continuous symmetric and coercive , K is a convex , closed set $K \subset V$ and $f : V \to R$ a linear application then the variational inequality problem:

$$(P1) Find\_u \in K : a(u, v-u) \geq f(v-u) \forall v \in K \tag{2}$$

is equivalent with the problem of minimization:

$$(P2) Find\_u \in K, J(u) \leq J(v) \forall v \in K . \tag{3}$$

Definition . A family V_h of Hilbert spaces with finite dimension is named interne approximation for V if:

$$\forall h > 0, V_h \subset V \tag{4}$$

and exists a space $\overline{V}$ dense in V :

$$\forall v \in \overline{V}, \exists v_h \in V_h, v_h \to v, h \to 0 . \tag{5}$$

Definition 3. A family of sets K_h is named interne approximation for K if all the follows is true: K_h is convex and closed , $K_h \subset V_h$,

$$\forall v \in K, \exists v_h \in K_h : v_h \to v, h \to 0 \tag{6}$$

and:

$$if \;\; v_h \in K_h v_h \to v \;\; weak \;\; in \;\; V \;\; then \;\; v \in K . \tag{7}$$

Using the interne approximations K_h for K and V_h for V we obtain the problem in finite dimension:

$$(P3) Find \;\; u_h \in K_h : a(u_h, v-u_h) \geq f(v-u_h) \forall v \in K_h . \tag{8}$$

The following theorem describes the hypothesis that is necessaries for $u_h \to u$.

Theorem 1. Let be K_h an interne approximation for K and V_h an interne approximation for V .Let be $a : V \times V \to R$ a bi-linear, continuous symmetric and coercive form and $f : V \to R$ a linear application .If u is the solution of problem (P1) and u_h is the solution of problem (P3) then $u_h \to u$ in V

The demonstration of this theorem can be found in [6].

We consider a domain $\Omega \subset R^d$ with a sufficiently regular boundary Γ.

If $V = H^1(\Omega)$ or $V = L^2(\Omega)$ then an interne approximation for V is obtained using the finite element method described in the following:

A triangulation of Ω is obtained by subdividing Ω into a set $T_h = \{E_1, E_2, ...E_m\}$.

The polyhedrons $E_1, E_2, ..., E_m$ are named the reels elements of triangulation and satisfied the following conditions:

$$\begin{aligned} &\overline{\Omega} = \cup E_i \\ &\text{int}(E_i) \neq \Phi, \forall i \in \{1,2,..,m\} \\ &\text{int}(E_i) \cap \text{int}(E_j) = \Phi, \forall i \neq j \end{aligned} \tag{9}$$

and if $F = E_i \cap E_j$ then F is a common face ,vertex or edge.

We introduce the mesh parameter $h = \max(meas(E_i)), i \in \{1,2,..,m\}$.

The most used polyhedron is: for d=2: triangles or parallelograms ;for d=3: thetraedres or parallelepipeds. We consider that the each reel element E can be obtained using a reference element $\overline{E}$ and an invertible ,affine transformation T_E:

$$E = T_E(\overline{E}), T_E(\hat{x}, \hat{y}, \hat{z}) = (B_E \cdot (\hat{x}, \hat{y}, \hat{z})^T + b_E)^T \tag{10}$$

with B_E an invertible matrix.

In this article we use the reference element $[0,1]^d$ named d-unity cube.

Let be Q_l the linear space of polynomials functions with grade l in each variables .The dimension of this space is:

$$\dim(Q_l) = (l+1)^d. \tag{11}$$

We use the following approximation for $V = H^1(\Omega)$:

$$V_h = V_h^l = \{v_h \in C^0(\overline{\Omega}) \big| v_h\big|_E \circ T_E^{-1} \in Q_l \forall E \in T_h\} \tag{12}$$

named the space of parallelepiped finite element.

The following proposition assures the inclusion:

$$V_h \subset H^1(\Omega). \tag{13}$$

The demonstration is not presented in this work.

Proposition 2. Let be $\Omega \subset R^d$ an open set and T_h a triangulation for Ω.A function $f : \Omega \to R$ is in the space $H^1(\Omega)$ if and only if :

$$\begin{aligned} &a) v\big|_E \in H^1(E) \forall E \in T_h \\ &b) \forall F = E_1 \cap E_2, E_1, E_2 \in T_h, trace_F(v\big|_{E_1}) = trace_F(v\big|_{E_2}) \end{aligned}. \tag{14}$$

Observation 1.F is a common face.

A polynomial function $P \in Q_l$ is well defined in the reference element $\hat{E} = [0,1]^d$ if are known the values of this in $n = \dim(Q_l) = (l+1)^d$ points named the liberty grades .

For $l = 1$ and $d = 3$ those points are:

$$A_1(0,0,0), A_2(1,0,0), A_3(1,1,0), A_4(0,1,0), A_5(0,0,1), A_6(1,0,1), A_7(1,1,1), A_8(0,1,1) \tag{15}$$

the vertices of the cube.

In this case the dimension of the space V_h described bellow is equal with the number of all nodes in triangulation T_h.Let being $\Sigma_h = \{N_1, N_2, ..., N_p\}$ the set of all the nodes of T_h.

Definition 4. The functions $\varphi_1,..,\varphi_p \in V_h$ are the shape functions for the triangulation T_h if:

$$\varphi_i(N_j) = \delta_{ij} \forall i, j \in \{1,2,..,p\}. \tag{16}$$

The shape functions for the reference element can be found in [9]:

Proposition 3. The shape functions for the reference element $[0,1]^3$ is:

$$\begin{aligned}
&\omega_1(\xi,\eta,\zeta) = -(\xi-1)(\eta-1)(\zeta-1); \omega_2(\xi,\eta,\zeta) = \xi(\eta-1)(\zeta-1) \\
&\omega_3(\xi,\eta,\zeta) = -\xi\eta(\zeta-1); \omega_4(\xi,\eta,\zeta) = (\xi-1)\eta(\zeta-1) \\
&\omega_5(\xi,\eta,\zeta) = (\xi-1)(\eta-1)\zeta; \omega_6(\xi,\eta,\zeta) = -\xi(\eta-1)\zeta \\
&\omega_7(\xi,\eta,\zeta) = \varsigma\eta\zeta; \omega_8(\xi,\eta,\zeta) = -(\xi-1)\eta\zeta
\end{aligned} \tag{17}$$

having the following property:

$$\omega_i(A_j) = \delta_{ij} \forall i, j \in \{1,2,..,8\}. \tag{18}$$

Let be a real element $E = P_1P_2P_3P_4P_5P_6P_7P_8 \in T_h$ having the coordinates $P_i(x_i, y_i, z_i)$.

Proposition 4. The invertible affine application that transform the reference element $\overline{E}$ into the real element $E = P_1P_2P_3P_4P_5P_6P_7P_8$ and $T_E(A_i) = P_i$ is:

$$(T_E((\xi,\eta,\zeta)))^T = \begin{pmatrix} x_2 - x_1 & x_3 - x_2 & x_5 - x_1 \\ y_2 - y_1 & y_3 - y_2 & y_5 - y_1 \\ z_2 - z_1 & z_3 - z_2 & z_5 - z_1 \end{pmatrix} \begin{pmatrix} \xi \\ \eta \\ \zeta \end{pmatrix} + \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}. \tag{19}$$

Let find now the global shape functions $\varphi_1, \varphi_2, .., \varphi_p \in V_h$.Fixing a node $N_i \in \Sigma_h$,exists one or more real elements $E_{l1}, E_{l2}, ...E_{lr}$ with $N_i \in E_{lj} \forall j \in \{1,2..,r\}$.

Proposition 5. The shape function φ_i is:

$$\varphi_i(x,y,z) = \begin{cases} \omega_{j1} \circ T_{l1}^{-1}(x,y,z), (x,y,z) \in E_{l1}, T_{l1}(A_{j1}) = N_i \\ \omega_{j2} \circ T_{l2}^{-1}(x,y,z), (x,y,z) \in E_{l2}, T_{l2}(A_{j2}) = N_i \\ .... \\ \omega_{jr} \circ T_{lr}^{-1}(x,y,z), (x,y,z) \in E_{lr}, T_{lr}(A_{jr}) = N_i \\ 0, (x,y,z) \notin E_{lj} \forall j \in \{1,2,..,r\} \end{cases} \tag{20}$$

with the notation $T_{l1} = T_{E_{l1}}$.

$\{\varphi_1, \varphi_2, .., \varphi_p\}$ is a base for the space V_h:

$$\forall v \in V_h, \exists v^i \in R, v(x,y,z) = \Sigma v^i \varphi_i(x,y,z) \tag{21}$$

and $v^i = v(N_i)$ is the value of the function in the points that correspond at the node N_i.

An Example of a Variational Inequality Obtained from a Boundary Elasto-plastic Problem

Let be a orthonormate base $\{i_1, i_2, i_3\}$.We consider a solid body ; $\Omega \subset R^3$ is the domain occupied at the moment t_0 and Ω_t is the domain occupied at the moment of time t.The properties of the material is the following: material elasto-plastic isotropic homogeneous with small deformations and isotropic hardening .The constitutive assumption is presented in [1] ,[3] and can be found in the article "*A numerical solution for a problem of elasto-plasticity-small deformations*" .In this article we have only the isotropic hardening ; the internal variable α that describe the cinematic hardening is null over the entire process. We neglect the body forces. The following notation will be used: σ is the Cauchy stress tensor for small deformations, u is displacement, ∇u is the gradient of u, $\varepsilon = \frac{1}{2}(\nabla u + (\nabla u)^T)$ is the tensor of total deformation, K and G is the constants of linear elasticity, P is a known function, $tr\varepsilon = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}$, $\sigma' = \sigma - \frac{1}{3}(tr\sigma)I$ is the deviator of σ.

κ is the scalar variable who describe the isotropic hardening, $v = \dot{u}$ is the velocity,

$\lambda = \frac{1}{h_c} < \beta > H(F)$ is the plastic multiplier , β is the plastic factor.

The equilibrium equation is:

$$div(\sigma) = 0 \tag{22}$$

and we suppose the following boundary conditions:

$$\begin{aligned} \sigma(x,t)n|\Gamma_1 &= g(x,t) \\ u(x,t)|\Gamma_2 &= U(x,t) \end{aligned}. \tag{23}$$

In (26) the functions f and U are known and $\partial\Omega = \Gamma = \Gamma_1 \cup \Gamma_2, \Gamma_1 \cap \Gamma_2 = \Phi$.

The set of all the points that verifies the yield condition $F(\sigma, \kappa) = 0$ at the moment t is Ω_t^p (the plastic domain) and $\Omega_t^e = \Omega_t - \Omega_t^p$ is the elastic domain. Let be:

$$V_{ad} = \{v : \Omega_t \to R^3 | v|\Gamma_{2t} = \dot{U}\} \tag{24}$$

the set of all velocities admissible at the moment of time t and

$$K = \{(w,\delta) : \Omega_t \to R^3 \times R, w \in V_{ad}, \delta \geq 0, \delta(x) = 0 \forall x \in \Omega^e\}. \tag{25}$$

This set is closed and convex. The values of h_c is:

$$h_c = \frac{2}{3} P(\kappa) P'(\kappa) > 0 \tag{26}$$

and is known. We suppose that $\sigma, \kappa, \Omega_t, \Omega_t^p$ is known at the moment of time *t*.

The following theorem represents the formulation with variational inequality for the problem described bellow. The velocity field v and the plastic factor β are the unknown of the problem.

Theorem 2. At every moment of time *t* the velocity field v and the plastic factor β satisfy the following relation:

$$\begin{aligned}&\int_{\Omega}[2G\dot{\varepsilon} + (K - \frac{2}{3}G)(tr\dot{\varepsilon})I] \cdot (\nabla w - \nabla v)dx - \int_{\Omega^p} \beta \frac{3G}{h_c P(\kappa)} \sigma' \cdot (\nabla w - \nabla v)dx - \\ &\int_{\Omega^p} (\delta - \beta)\frac{1}{h_c}(2G\sigma' \cdot \dot{\varepsilon})dx + \int_{\Omega^p} \beta(\delta - \beta)\left(\frac{1}{h_c} + \frac{1}{h_c^2}\frac{3G}{P(\kappa)}\sigma' \cdot \sigma'\right)dx \geq \int_{\Gamma 1} g \cdot (w - v)da\end{aligned} \tag{27}$$

for all $(w, \delta) \in K$.

We use the following notations:

$$f(w) = \int_{\Gamma 1} g \cdot w da, \tag{28}$$

$$\begin{aligned}a[(v,\beta),(w,\delta)] = &\int_{\Omega}[2G\dot{\varepsilon} + (K - \frac{2}{3}G)(tr\dot{\varepsilon})I] \cdot (\nabla w)dx - \int_{\Omega^p} \beta \frac{3G}{h_c P(\kappa)} \sigma' \cdot (\nabla w)dx - \\ &\int_{\Omega^p} (\delta)\frac{1}{h_c}(2G\sigma' \cdot \dot{\varepsilon})dx + \int_{\Omega^p} \beta\delta\left(\frac{1}{h_c} + \frac{1}{h_c^2}\frac{3G}{P(\kappa)}\sigma' \cdot \sigma'\right)dx\end{aligned} \quad .(29)$$

With this notation we have to solve the variational inequality: find $(v, \beta) \in K$ so that

$$a[(v,\beta),(w,\delta) - (v,\beta)] \geq f[(w,\delta) - (v,\beta)] \forall (w,\delta) \in K. \tag{30}$$

Observation: a is bi-linear and symmetric on $V = (H^1(\Omega_t)^3 \times L^2(\Omega_t))$.

We obtain for a the following representation:

$$a[(v,\beta),(w,\delta)] = \iiint_{\Omega_t} M \begin{pmatrix} \partial v_1/\partial x \\ \partial v_1/\partial y \\ \partial v_1/\partial z \\ \partial v_2/\partial x \\ \partial v_2/\partial y \\ \partial v_2/\partial z \\ \partial v_3/\partial x \\ \partial v_3/\partial y \\ \partial v_3/\partial z \\ \beta \end{pmatrix} \cdot \begin{pmatrix} \partial w_1/\partial x \\ \partial w_1/\partial y \\ \partial w_1/\partial z \\ \partial w_2/\partial x \\ \partial w_2/\partial y \\ \partial w_2/\partial z \\ \partial w_3/\partial x \\ \partial w_3/\partial y \\ \partial w_3/\partial z \\ \delta \end{pmatrix}^T dx. \tag{31}$$

The matrix $M = M(x, y, z)$ is not constant but is known at the fixed time *t*.

$$
\begin{aligned}
&M_{11} = M_{55} = M_{99} = \frac{4}{3}G + K; M_{15} = M_{19} = M_{51} = M_{59} = M_{91} = M_{95} = K - \frac{2}{3}G;\\
&M_{22} = M_{24} = M_{33} = M_{37} = M_{42} = M_{44} = M_{66} = M_{68} = M_{73} = M_{77} = M_{86} = M_{88} = G\\
&M_{1,10} = M_{10,1} = -T\sigma'_{11}H(F); M_{2,10} = M_{10,2} = M_{4,10} = M_{10,4} = -T\sigma'_{12}H(F);\\
&M_{6,10} = M_{10,6} = M_{8,10} = M_{10,8} = -T\sigma'_{23}H(F); M_{4,10} = M_{10,4} = M_{7,10} = M_{10,7} =\\
&\qquad = -T\sigma'_{13}H(F);\\
&M_{5,10} = M_{10,5} = -T\sigma'_{22}H(F); M_{9,10} = M_{10,9} = -T\sigma'_{33}H(F); M_{10,10} =\\
&\qquad = \frac{1}{h_c} + \frac{1}{h_c^2}\frac{3G}{P(\kappa)}\sigma' \cdot \sigma'
\end{aligned} \tag{32}
$$

with $T = \dfrac{3G}{h_c P(\kappa)}$, $H(\cdot)$ the Heaviside function and F the yield function. *M* is symmetric.

Let be $\Omega_t = [0,l] \times [0,2L] \times [0,c]$ and the nodes

$$
\begin{aligned}
&\{N_1(0,0,0), N_2(l,0,0), N_3(l,L,0), N_4(0,L,0), N_5(l,2L,0), N_6(0,2L,0),\\
&N_7(0,2L,c), N_8(l,2L,c), N_9(l,L,c), N_{10}(0,L,c), N_{11}(0,0,c), N_{12}(l,0,c)\}
\end{aligned}
$$

of a simple triangulation $T_h = \{E_1, E_2\}$ with $E_1 = N_1N_2N_3N_4N_{11}N_{12}N_9N_{10}$ and

$$E_2 = N_4N_3N_5N_6N_{10}N_9N_8N_7.$$

For *l*=1, *L*=3, *c*=1 we obtain the following shape functions using a Math-lab program:

$$\varphi_i(x,y,z) = \begin{cases} \text{shape1(i)}, (x,y,z) \in E_1 \\ \text{shape2(i)}, (x,y,z) \in E_2 \end{cases}, \forall i \in \{1,2,..12\} \tag{33}$$

and the vectors shape1,shape2 are:

$$
\begin{aligned}
&\text{shape1} = [(-x+1)((1/3)y-1)(z-1), x((1/3)y-1)(z-1), -(1/3)xy(z-1),\\
&(1/3)(x-1)y(z-1), 0, 0, 0, 0, (1/3)xyz,\\
&1/3(-x+1)yz, (x-1)((1/3)y-1)z, -x((1/3)y-1)z]
\end{aligned} \tag{34}
$$

$$
\begin{aligned}
&\text{shape2} = [0, 0, x((1/3)y-2)(z-1), (-x+1)((1/3)y-2)(z-1),\\
&-x((1/3)y-1)(z-1), (x-1)((1/3)y-1)(z-1), (-x+1)((1/3)y-1)z, .\\
&x((1/3)y-1)z, -x((1/3)y-2)z, (x-1)((1/3)y-2)z, 0, 0]
\end{aligned} \tag{35}
$$

The interne approximation for V is $V_h = (Sp\{\varphi_1, \varphi_2, ...\varphi_{12}\})^4$ .Let be:

$$(v = (v_1, v_2, v_3), \beta) \in V_h, (w = (w_1, w_2, w_3), \delta) \in V_h. \tag{36}$$

Using the shape functions we can write:

$$
\begin{aligned}
&v_1 = v_1^1\varphi_1 + ... + v_1^{12}\varphi_{12}; v_2 = v_2^1\varphi_1 + ... + v_2^{12}\varphi_{12};\\
&v_3 = v_3^1\varphi_1 + ... + v_3^{12}\varphi_{12}; \beta = \beta^1\varphi_1 + .. + \beta^{12}\varphi_{12}
\end{aligned} \tag{37}
$$

and similar for (w,δ).In $V_h \times V_h$ the bi-linear form a became a bi-linear form in the components:

$$(v_1^1,..,v_1^{12},v_2^1,..,v_2^{12},v_3^1,..,v_3^{12},\beta^1,..,\beta^{12}),(w_1^1,..,w_1^{12},w_2^1,..,w_2^{12},w_3^1,..,w_3^{12},\delta^1,..,\delta^{12}) \quad (38)$$

with the matrix associated , $\widetilde{M}$ depending on the domain Ω_t.

The linear application takes the following form on V_h:

$$\begin{aligned} f(w) &= (f_1,f_2,..,f_{36})\cdot(w_1^1,..,w_1^{12},w_2^1,..,w_2^{12},w_3^1,..,w_3^{12})^T = \\ &= (f_1,f_2,..,f_{48})(w_1^1,..,w_1^{12},w_2^1,..,w_2^{12},w_3^1,..,w_3^{12}.\delta^1,..,\delta^{12})^T,. \\ f_{37} &= f_{38}..=f_{48}=0 \end{aligned} \quad (39)$$

The approximation for the convex K is:

$$\begin{aligned} K_h = \{(w_1^1,..,w_1^{12},w_2^1,..,w_2^{12},w_3^1,..,w_3^{12}.\delta^1,..,\delta^{12}) \in R^{48} \,\big|\, w_j^i = \dot{U}_j(N_i)\forall i, N_i \in \Gamma_{2t} \\ and\ \delta^i = 0\ \forall i\ with\ N_i \in \Omega_t^e\ and\ \delta^i \geq 0 \forall i\ with\ N_i \in \Omega_t^p\} \end{aligned}. \quad (40)$$

Proposition 6. The problem in finite dimension is:

Find $(v_1^1,..,v_1^{12},v_2^1,..,v_2^{12},v_3^1,..,v_3^{12},\beta^1,..,\beta^{12}) \in K_h$ with the property:

$$\begin{aligned} &\widetilde{M}(v_1^1,..,v_1^{12},v_2^1,..,v_2^{12},v_3^1,..,v_3^{12},\beta^1,..,\beta^{12})^T \cdot \\ &((w_1^1,..,w_1^{12},w_2^1,..,w_2^{12},w_3^1,..,w_3^{12}.\delta^1,..,\delta^{12})^T - \\ &-(v_1^1,..,v_1^{12},v_2^1,..,v_2^{12},v_3^1,..,v_3^{12},\beta^1,..,\beta^{12})^T) \geq \\ &(f_1,...f_{48})\cdot((w_1^1,..,w_1^{12},w_2^1,..,w_2^{12},w_3^1,..,w_3^{12}.\delta^1,..,\delta^{12})^T - \\ &-(v_1^1,..,v_1^{12},v_2^1,..,v_2^{12},v_3^1,..,v_3^{12},\beta^1,..,\beta^{12})^T) \\ &\forall(w_1^1,..,w_1^{12},w_2^1,..,w_2^{12},w_3^1,..,w_3^{12}.\delta^1,..,\delta^{12}) \in K_h \end{aligned} \quad (41)$$

If $\widetilde{M}$ is positive definite then the bellow problem is equivalent with the minimization on K_h of the application *J*: $J((w,\delta)) = \frac{1}{2}a[(w,\delta),(w,\delta)] - f((w,\delta))$.

The dimension used is high: 4*p* for p numbers of nodes so for solving this problem in finite dimension we can use optimization algorithms like gradient and projected gradient with the possibility to make computer programs for this.

But first are necessaries to fix the mathematical cadre and the conditions for existence and uniqueness of solution and establish the conditions for convergence of algorithms. Such information can be found in [6] and [4] .Without this complicated discussions exists the possibility of no convergence of the algorithm or a no convergence of the FEM.

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O aproximație internă pentru o problemă de inegalitate variațională

Rezumat

În acest articol descriem o modalitate de a transforma o inegalitate variațională în dimensiune infinită într-o problemă în dimensiune finită. Utilizăm acest concept teoretic pentru o problemă de elasto-plasticitate şi pentru un domeniu simplu. Se utilizează un program Matlab pentru a calcula integralele triple care apar în algoritmul metodei elementului finit.