On the Points of Weak m-Continuity and Weak m-Discontinuity

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Abstract
In the present paper, we define the notion of weakly m-continuous functions at a point of the domain and obtain some characterizations of these functions. Furthermore, we characterize the set of all points at which the function is not weakly m-discontinuous.

Key words: *m-structure, m-space, weak m-continuity, weak m-discontinuity*.

Introduction

Semi-open sets, preopen sets, α-sets, and β-open sets play an important role in the researches of generalizations of continuity in topological spaces. By using these sets, several authors introduced and studied various types of weak forms of continuity: semi-continuity or quasi-continuity, precontinuity, α-continuity, and β-continuity and other forms.

In 1961, Levine [13] introduced the concept of weakly continuous functions. In [7], [13], [21], [22], [30] and other papers, the authors studied many properties of weakly continuous functions. In 1963, Levine [14] introduced the concept of semi-continuous functions. Neubrunnová [19] showed that semi-continuity is equivalent to quasi-continuity due to Marcus [16]. In 1973, Popa and Stan [38] introduced and studied the concept of weakly quasi-continuous functions. Weak quasi-continuity is implied by both quasi-continuity and weak continuity which are independent of each other. It is shown in [23] that weak quasi-continuity is equivalent to semi-weak continuity in the sense of Costovici [4] and weak-semi-continuity in the sense of Arya and Bhamini [3] and Kar and Bhattacharyya [12]. Many properties of weakly quasi-continuous functions are studied in [3], [11], [12], [23], [27], [28], [31] and other papers. In 1982, Janković [10] introduced the notion of almost weakly continuous functions. It is shown in [32] that almost weak continuity is equivalent to quasi precontinuity due to Paul and Bhattacharyya [29]. Many properties of almost weakly continuous functions are studied in [9], [28], [29] and [32]. In 1987, Noiri [24] introduced the notion of weakly α-continuous functions. Several properties of weakly α-continuous functions are studied in [24], [39] and [40]. The present authors [33] introduced and studied weakly β-continuous functions which are equivalent to weakly semi-precontinuous functions due to Ghosh and Bhattacharyya [8]. Some properties of weakly β-continuous functions are studied in [33], [8], [25], and [26]. In [35], the present authors introduced the notion of m-continuous functions as a function from a set satisfying some minimal conditions into a topological space. In [36], the present authors obtained some charactrizations for functions which are m-continuous at a point of the domain and characterized the set of all points at which the function is not m-continuous.

In this paper, we define the notion of weakly m-continuous functions at a point of the domain and obtain some characterizations of these functions and characterize the set of all such points at which the function is not weakly m-continuous.

Preliminaries

Let (X,τ) be a topological space and A a subset of X. The closure of A and the interior of A are denoted by Cl(A) and Int(A), respectively. A subset A is said to be *regular open* (resp. *regular closed*) if Int(Cl(A)) = A (resp. Cl(Int(A)) = A).

Definition 1 Let (X,τ) be a topological space. A subset A of X is said to be α-*open* [20] (resp. *semi-open* [14], *preopen* [17], β-*open* [1]) if $A \subset \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(A)))$ (resp. $A \subset \mathrm{Cl}(\mathrm{Int}(A))$, $A \subset \mathrm{Int}(\mathrm{Cl}(A))$, $A \subset \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(A)))$).

The family of all α-open (resp. semi-open, preopen, β-open) sets in (X,τ) is denoted by $\alpha(X)$ (resp. SO(X), PO(X), $\beta(X)$).

Definition 2 Let (X,τ) be a topological space. A subset A of X is said to be α-*closed* [18] (resp. *semi-closed* [5], *preclosed* [17], β-*closed* [1]) if the complement of A is α-open (resp. semi-open, preopen, β-open).

Definition 3 Let (X,τ) be a topological space and A a subset of X. The intersection of all α-closed (resp. semi-closed, preclosed, β-closed) sets of X containing A is called the α-*closure* [18] (resp. *semi-closure* [5], *preclosure* [6], β-*closure* [2]) of A and is denoted by αCl(A) (resp. sCl(A), pCl(A), ${}_{\beta}$Cl(A)).

Definition 4 Let (X,τ) be a topological space and A a subset of X. The union of all α-open (resp. semi-open, preopen, β-open) sets of X contained in A is called the α-*interior* [18] (resp. *semi-interior* [5], *preinterior* [6], β-*interior* [2]) of A and is denoted by αInt(A) (resp. sInt(A), pInt(A), ${}_{\beta}$Int(A)).

A point $x \in X$ is called a θ-*cluster* point of a subset A of X [41] if $\mathrm{Cl}(V) \cap A \neq \emptyset$ for every open set V containing x. The set of all θ-cluster points of A is called the θ-*closure* of A and is denoted by $\mathrm{Cl}_\theta(A)$. If $A = \mathrm{Cl}_\theta(A)$, then A is said to be θ-closed [41]. The complement of a θ-closed set is said to be θ-open. The union of all θ-open sets contained in A is called the θ-interior of A and is denoted by $\mathrm{Int}_\theta(A)$. It is shown in [41] that $\mathrm{Cl}_\theta(V) = \mathrm{Cl}(V)$ for every open set V of X and $\mathrm{Cl}_\theta(A)$ is closed in X for each subset A of X.

Throughout the present paper, (X,τ) and (Y,σ) always denote topological spaces.

Definition 5 A function $f : (X,\tau) \to (Y,\sigma)$ is said to be *weakly continuous* [13] (resp. *weakly semi-continuous* [3], [4], [12] or *weakly quasi-continuous* [38], *almost weakly continuous* [10] or *quasi-precontinuous* [28], *weakly* α-*continuous* [24], *weakly* β-*continuous* [33] or *weakly semi-precontinuous* [8]) at a point $x \in X$ if for each open set V containing $f(x)$, there exists an open (resp. semi-open, preopen, α-open, β-open) set U of X containing x such that $f(U) \subset \mathrm{Cl}(V)$,

A function $f : (X,\tau) \to (Y,\sigma)$ is said to be *weakly continuous* (resp. *weakly semi-continuous* or *weakly quasi-continuous*, *almost weakly continuous* or *quasi-precontinuous*, *weakly* α-*continuous* , *weakly* β-*continuous* or *weakly semi-precontinuous*) if it has this property at every point of X.

Weakly m-Continuous Functions at a Point $x \in X$

Definition 6 A subfamily m_X of the power set $\mathcal{P}(X)$ of a nonempty set X is called a *minimal structure* (briefly *m-structure*) on X [34], [35] if $\emptyset \in m_X$ and $X \in m_X$.

By (X, m_X), we denote a nonempty subset X with a minimal structure m_X on X and call it an *m-space*. Each member of m_X is said to be m_X-open (or briefly m-open) and the complement of an m_X-open set is said to be m_X-closed (or briefly m-closed).

Remark 1 Let (X, τ) be a topological space. Then the families τ, SO(X), PO(X), $\alpha(X)$, and $\beta(X)$ are all m-structures on X.

Definition 7 Let (X, m_X) be an m-space. For a subset A of X, the *m_X-closure* of A and the *m_X-interior* of A are defined in [15] as follows:
(1) $\mathrm{mCl}(A) = \cap\{F : A \subset F, X - F \in m_X\}$,
(2) $\mathrm{mInt}(A) = \cup\{U : U \subset A, U \in m_X\}$.

Remark 2 Let (X, τ) be a topological space and A a subset of X. If $m_X = \tau$ (resp. SO(X), PO(X), $\alpha(X)$, $\beta(X)$), then we have
(1) $\mathrm{mCl}(A) = \mathrm{Cl}(A)$ (resp. $\mathrm{sCl}(A)$, $\mathrm{pCl}(A)$, $\alpha\mathrm{Cl}(A)$, ${}_\beta\mathrm{Cl}(A)$),
(2) $\mathrm{mInt}(A) = \mathrm{Int}(A)$ (resp. $\mathrm{sInt}(A)$, $\mathrm{pInt}(A)$, $\alpha\mathrm{Int}(A)$, ${}_\beta\mathrm{Int}(A)$) .

Lemma 1 (Maki et al. [15]). *Let (X, m_X) be an m-space. For subsets A and B of X, the following hold:*
(1) $\mathrm{mCl}(X - A) = X - (\mathrm{mInt}(A))$ *and* $\mathrm{mInt}(X - A) = X - (\mathrm{mCl}(A))$,
(2) If $(X - A) \in m_X$, then $\mathrm{mCl}(A) = A$ *and if $A \in m_X$, then* $\mathrm{mInt}(A) = A$,
(3) $\mathrm{mCl}(\emptyset) = \emptyset$, $\mathrm{mCl}(X) = X$, $\mathrm{mInt}(\emptyset) = \emptyset$ *and* $\mathrm{mInt}(X) = X$,
(4) If $A \subset B$, then $\mathrm{mCl}(A) \subset \mathrm{mCl}(B)$ *and* $\mathrm{mInt}(A) \subset \mathrm{mInt}(B)$,
(5) $A \subset \mathrm{mCl}(A)$ *and* $\mathrm{mInt}(A) \subset A$,
(6) $\mathrm{mCl}(\mathrm{mCl}(A)) = \mathrm{mCl}(A)$ *and* $\mathrm{mInt}(\mathrm{mInt}(A)) = \mathrm{mInt}(A)$.

Lemma 2 (Noiri and Popa [34]). *Let (X, m_X) be an m-space and A a subset of X. Then $x \in \mathrm{mCl}(A)$ if and only if $U \cap A \neq \emptyset$ for every $U \in m_X$ containing x.*

Definition 8 A function $f : (X, m_X) \to (Y, \sigma)$ is said to be *m-continuous* [35] (resp. *weakly m-continuous*) at a point $x \in X$ if for each open set V of Y containing $f(x)$, there exists $U \in m_X$ containing x such that $f(U) \subset V$ (resp. $f(U) \subset \mathrm{Cl}(V)$).
The function f is said to be m-continuous (resp. weakly m-continuous) if it has this property at each $x \in X$.

Theorem 1 *For a function $f : (X, m_X) \to (Y, \sigma)$, the following properties are equivalent:*
(1) f is weakly m-continuous at $x \in X$;
(2) for every open set V of Y with $x \in f^{-1}(V)$, $x \in \mathrm{mInt}(f^{-1}(\mathrm{Cl}(V)))$;
(3) for every closed set F of Y and $x \in \mathrm{mCl}(f^{-1}(\mathrm{Int}(F)))$, $x \in f^{-1}(F)$;
(4) for every subset B of Y with $x \in \mathrm{mCl}(f^{-1}(\mathrm{Int}(\mathrm{Cl}(B))))$, $x \in f^{-1}(\mathrm{Cl}(B))$;
(5) for every subset B of Y and $x \in f^{-1}(\mathrm{Int}(B))$, $x \in \mathrm{mInt}(f^{-1}(\mathrm{Cl}(\mathrm{Int}(B))))$;
(6) for every open set V of Y with $x \in \mathrm{mCl}(f^{-1}(V))$, $x \in f^{-1}(\mathrm{Cl}(V))$.

Proof. (1) $\Rightarrow$ (2): Let V be an open set of Y such that $x \in f^{-1}(V)$. Then $f(x) \in V$. There exists $U \in m_X$ containing x such that $f(U) \subset \mathrm{Cl}(V)$. Thus, we obtain $x \in U \subset f^{-1}(\mathrm{Cl}(V))$. This implies that $x \in \mathrm{mInt}(f^{-1}(\mathrm{Cl}(V)))$.
(2) $\Rightarrow$ (3): Let F be any closed set of Y. Suppose that $x \notin f^{-1}(F)$. Then $Y - F$ is open in Y and $x \in X - f^{-1}(F) = f^{-1}(Y - F)$. By (2) and Lemma 1, we have

$$x \in \mathrm{mInt}(f^{-1}(\mathrm{Cl}(Y-F))) = \mathrm{mInt}(f^{-1}(Y-\mathrm{Int}(F))) = \mathrm{mInt}(X - f^{-1}(\mathrm{Int}(F))) = X - (\mathrm{mCl}(f^{-1}(\mathrm{Int}(F))).$$

Therefore, we obtain $x \notin \mathrm{mCl}(f^{-1}(\mathrm{Int}(F)))$.
(3) $\Rightarrow$ (4): Let B be any subset of Y. Then $\mathrm{Cl}(B)$ is closed in Y and by (3) we have that if $x \in \mathrm{mCl}(f^{-1}(\mathrm{Int}(\mathrm{Cl}(B))))$ then $x \in f^{-1}(\mathrm{Cl}(B))$.
(4) $\Rightarrow$ (5): Let B be any subset of Y and $x \in f^{-1}(\mathrm{Int}(B))$. Then we have $x \in f^{-1}(\mathrm{Int}(B)) = X - f^{-1}(\mathrm{Cl}(Y-B))$. Then $x \notin f^{-1}(\mathrm{Cl}(Y-B))$ and by (4) $x \in X - \mathrm{mCl}(f^{-1}(\mathrm{Int}(\mathrm{Cl}(Y-B)))) = \mathrm{mInt}(f^{-1}(\mathrm{Cl}(\mathrm{Int}(B))))$.
(5) $\Rightarrow$ (6): Let V be any open set of Y. Suppose that $x \notin f^{-1}(\mathrm{Cl}(V))$. Then $f(x) \notin \mathrm{Cl}(V)$ and there exists an open set W containing $f(x)$ such that $W \cap V = \emptyset$; hence $\mathrm{Cl}(W) \cap V = \emptyset$. By (5), we have $x \in \mathrm{mInt}(f^{-1}(\mathrm{Cl}(W)))$ and hence there exists $U \in m_X$ such that $x \in U \subset f^{-1}(\mathrm{Cl}(W))$. Since $\mathrm{Cl}(W) \cap V = \emptyset, U \cap f^{-1}(V) = \emptyset$ and by Lemma 2 $x \notin \mathrm{mCl}(f^{-1}(V))$. Therefore, if $x \in \mathrm{mCl}(f^{-1}(V))$, then $x \in f^{-1}(\mathrm{Cl}(V))$.
(6) $\Rightarrow$ (1): Let $x \in X$ and V any open set of Y containing $f(x)$. Then, we have $x \in f^{-1}(V) \subset f^{-1}(\mathrm{Int}(\mathrm{Cl}(V))) = X - f^{-1}(\mathrm{Cl}(Y - \mathrm{Cl}(V)))$. By (6) $x \notin \mathrm{mCl}(f^{-1}(Y - \mathrm{Cl}(V)))$ and hence $x \in \mathrm{mInt}(f^{-1}(\mathrm{Cl}(V)))$. Therefore, there exists $U \in m_X$ such that $x \in U \subset f^{-1}(\mathrm{Cl}(V))$; hence $f(U) \subset \mathrm{Cl}(V)$. This shows that f is weakly m-continuous at $x \in X$.

Remark 3 Let (X,τ) and (Y,σ) be two topological spaces. We put $m_X = \tau$ (resp. $\mathrm{SO}(X)$, $\beta(X)$). Then a weakly m-continuous function $f : (X,m_X) \to (Y,\sigma)$ is weakly continuous (resp. weakly quasicontinuous, weakly β-continuous). Therefore, by Theorem 1 we obtain the results established in Theorem 4.1 of [13] (resp. Theorem 1 of [38], Theorem 1 of [33]).

Theorem 2 *For a function $f : (X,m_X) \to (Y,\sigma)$, the following properties are equivalent:*
(1) f is weakly m-continuous at $x \in X$;
(2) for every subset B of Y and $x \in \mathrm{mCl}(f^{-1}(\mathrm{Int}(\mathrm{Cl}_\theta(B)))), x \in f^{-1}(\mathrm{Cl}_\theta(B))$;
(3) for every subset B of Y and $x \in \mathrm{mCl}(f^{-1}(\mathrm{Int}(\mathrm{Cl}(B)))), x \in f^{-1}(\mathrm{Cl}_\theta(B))$;
(4) for every open set G of Y such that $x \in \mathrm{mCl}(f^{-1}(\mathrm{Int}(\mathrm{Cl}(G)))), x \in f^{-1}(\mathrm{Cl}(G))$;
(5) for every preopen set V of Y such that $x \in \mathrm{mCl}(f^{-1}(\mathrm{Int}(\mathrm{Cl}(V)))), x \in f^{-1}(\mathrm{Cl}(V))$;
(6) for every regular closed set K of Y and $x \in \mathrm{mCl}(f^{-1}(\mathrm{Int}(K))), x \in f^{-1}(K)$;
(7) for every β-open set G of Y and $x \in \mathrm{mCl}(f^{-1}(\mathrm{Int}(\mathrm{Cl}(G)))), x \in f^{-1}(\mathrm{Cl}(G))$;
(8) for every semi-open set G of Y and $x \in \mathrm{mCl}(f^{-1}(\mathrm{Int}(\mathrm{Cl}(G)))), x \in f^{-1}(\mathrm{Cl}(G))$;
(9) for every subset A of X and $x \in \mathrm{mCl}(A), f(x) \in \mathrm{Cl}_\theta(f(A))$;
(10) for every subset B of Y such that $x \in \mathrm{mCl}(f^{-1}(B)), x \in f^{-1}(\mathrm{Cl}_\theta(B))$.

Proof. (1) $\Rightarrow$ (2): Let B be any subset of Y. Then $\mathrm{Cl}_\theta(B)$ is closed in Y. Then by Theorem 1(3), if $x \in \mathrm{mCl}(f^{-1}(\mathrm{Int}(\mathrm{Cl}_\theta(B))))$, then $x \in f^{-1}(\mathrm{Cl}_\theta(B))$.

(2) $\Rightarrow$ (3): This is obvious since $\mathrm{Cl}(B) \subset \mathrm{Cl}_\theta(B)$ for every subset B.

(3) $\Rightarrow$ (4): This is obvious since $\mathrm{Cl}(G) = \mathrm{Cl}_\theta(G)$ for every open set G.

(4) $\Rightarrow$ (5): Let $V \in \mathrm{PO}(Y)$ and $x \in \mathrm{mCl}(f^{-1}(\mathrm{Int}(\mathrm{Cl}(V))))$. Then we have $V \subset \mathrm{Int}(\mathrm{Cl}(V))$ and $\mathrm{Cl}(V) = \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(V)))$. Now, set $G = \mathrm{Int}(\mathrm{Cl}(V))$, then G is open in Y and $\mathrm{Cl}(G) = \mathrm{Cl}(V)$. By (4) we have $x \in f^{-1}(\mathrm{Cl}(G))$ and hence $x \in f^{-1}(\mathrm{Cl}(V))$.

(5) $\Rightarrow$ (6): Let K be any regular closed set of Y and $x \in \mathrm{mCl}(f^{-1}(\mathrm{Int}(K)))$. Then we have $\mathrm{Int}(K)$ is preopen in Y and $\mathrm{Int}(K) = \mathrm{Int}(\mathrm{Cl}(\mathrm{Int}(K)))$. Hence by (5) $x \in f^{-1}(\mathrm{Cl}(\mathrm{Int}(K))) = f^{-1}(K)$.

(6) $\Rightarrow$ (7): Let $G \in \beta(Y)$ and $x \in \mathrm{mCl}(f^{-1}(\mathrm{Int}(\mathrm{Cl}(G))))$. Then $G \subset \mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(G))))$. Since $\mathrm{Cl}(G)$ is regular closed, by (6) $x \in f^{-1}(\mathrm{Cl}(G))$.

(7) $\Rightarrow$ (8): This is obvious since $\mathrm{SO}(Y) \subset \beta(Y)$.

(8) $\Rightarrow$ (1): Let V be any open set of Y and $x \in \mathrm{mCl}(f^{-1}(V))$. Then, V is semi-open and $x \in \mathrm{mCl}(f^{-1}(\mathrm{Int}(\mathrm{Cl}(V))))$. By (8), $x \in f^{-1}(\mathrm{Cl}(V))$. It follows from Theorem 1 that f is weakly m-continuous at $x \in X$.

(1) $\Rightarrow$ (9): Let A be any subset of X. Let $x \in \mathrm{mCl}(A)$ and V be any open set of Y containing $f(x)$.

There exists $U \in m_X$ containing x such that $f(U) \subset \mathrm{Cl}(V)$. Since $x \in \mathrm{mCl}(A)$, by Lemma 2 we have $U \cap A \neq \emptyset$ and hence $\emptyset \neq f(U) \cap f(A) \subset \mathrm{Cl}(V) \cap f(A)$. Therefore, we have $f(x) \in \mathrm{Cl}_\theta(f(A))$.

(9) $\Rightarrow$ (10): Let B be any subset of Y and $x \in \mathrm{mCl}(f^{-1}(B))$. By (9), we have $f(x) \in \mathrm{Cl}_\theta(f(f^{-1}(B))) \subset \mathrm{Cl}_\theta(B))$ and hence $x \in f^{-1}(\mathrm{Cl}_\theta(B))$.

(10) $\Rightarrow$ (1): Let B be any subset of Y and $x \in \mathrm{mCl}(f^{-1}(\mathrm{Int}(\mathrm{Cl}(B))))$. By (10), we have $x \in f^{-1}(\mathrm{Cl}(\mathrm{Int}(\mathrm{Cl}(B)))) \subset f^{-1}(\mathrm{Cl}(B))$. It follows from Theorem 1(4) that f is weakly m-continuous at $x \in X$.

Remark 4 Let (X,τ) and (Y,σ) be two topological spaces. We put $m_X = \tau$. Then, a weakly m-continuous function $f : (X,m_X) \to (Y,\sigma)$ is weakly continuous. Therefore, by Theorem 2 we obtain the results established in Theorem 1 of [30].

Theorem 3 *For a function $f : (X,m_X) \to (Y,\sigma)$, the following properties are equivalent:*
(1) f is weakly m-continuous at $x \in X$;
(2) for every $V \in \mathrm{PO}(Y)$ with $x \in \mathrm{mCl}(f^{-1}(V))$, $x \in f^{-1}(\mathrm{Cl}(V))$;
(3) for every $V \in \mathrm{PO}(Y)$ with $x \in f^{-1}(V)$, $x \in \mathrm{mInt}(f^{-1}(\mathrm{Cl}(V)))$.

Proof. (1) $\Rightarrow$ (2): Let V be any preopen set of Y such that $x \in \mathrm{mCl}(f^{-1}(V))$. Suppose that $x \notin f^{-1}(\mathrm{Cl}(V))$. Then there exists an open set W containing $f(x)$ such that $W \cap V = \emptyset$. Hence we have $W \cap \mathrm{Cl}(V) = \emptyset$ and hence $\mathrm{Cl}(W) \cap \mathrm{Int}(\mathrm{Cl}(V)) = \emptyset$. Since V is preopen, $V \subset \mathrm{Int}(\mathrm{Cl}(V))$ and we have $V \cap \mathrm{Cl}(W) = \emptyset$. Since f is weakly m-continuous at $x \in X$ and W is an open set containing $f(x)$, there exists $U \in m_X$ containing x such that $f(U) \subset \mathrm{Cl}(W)$. Then $f(U) \cap V = \emptyset$ and hence $U \cap f^{-1}(V) = \emptyset$. This shows that $x \notin \mathrm{mCl}(f^{-1}(V))$. This is a contradiction. Therefore, we obtain $x \in f^{-1}(\mathrm{Cl}(V))$.

(2) $\Rightarrow$ (3): Let $V \in \mathrm{PO}(Y)$ and $x \in f^{-1}(V)$. Then, we have $f^{-1}(V) \subset f^{-1}(\mathrm{Int}(\mathrm{Cl}(V))) = X - f^{-1}(\mathrm{Cl}(Y - \mathrm{Cl}(V)))$. Therefore, $x \notin f^{-1}(\mathrm{Cl}(Y - \mathrm{Cl}(V)))$. Then by (2) $x \notin \mathrm{mCl}(f^{-1}(Y - \mathrm{Cl}(V)))$. Hence $x \in X - \mathrm{mCl}(f^{-1}(Y - \mathrm{Cl}(V))) = \mathrm{mInt}(f^{-1}(\mathrm{Cl}(V)))$.

(3) $\Rightarrow$ (1): This follows from Theorem 1 since every open set is preopen.

The Set of Points of Weak m-Discontinuity

In this section, we characterize the set of points at which a function $f : (X,m_X) \to (Y,\sigma)$ is not weakly m-continuous.

For a function $f : (X,m_X) \to (Y,\sigma)$, we define $D_{wmc}(f)$ as follows:

$$D_{wmc}(f) = \{x \in X : f \text{ is not weakly } m\text{-continuous at } x\}.$$

Theorem 4 *For a function $f : (X,m_X) \to (Y,\sigma)$, the following properties hold:*

$$\begin{aligned} D_{wmc}(f) &= \textstyle\bigcup_{G\in\sigma}\{f^{-1}(G) - \mathrm{mInt}(f^{-1}(\mathrm{Cl}(G)))\} \\ &= \textstyle\bigcup_{H\in\mathcal{F}}\{\mathrm{mCl}(f^{-1}(\mathrm{Int}(H))) - f^{-1}(H)\} \\ &= \textstyle\bigcup_{B\in\mathcal{P}(Y)}\{\mathrm{mCl}(f^{-1}(\mathrm{Int}(\mathrm{Cl}(B)))) - f^{-1}(\mathrm{Cl}(B))\} \\ &= \textstyle\bigcup_{B\in\mathcal{P}(Y)}\{f^{-1}(\mathrm{Int}(B)) - \mathrm{mInt}(f^{-1}(\mathrm{Cl}(\mathrm{Int}(B))))\} \\ &= \textstyle\bigcup_{G\in\sigma}\{\mathrm{mCl}(f^{-1}(G)) - f^{-1}(\mathrm{Cl}(G))\}, \end{aligned}$$

where $\mathcal{F}$ is the family of closed sets of (Y,σ).

Proof. We shall show only the first equality since the proofs of other are similar to the first.

Let $x \in D_{wmc}(f)$. By Theorem 1, there exists an open set V of Y such that $x \in f^{-1}(V)$ and $x \notin \mathrm{mInt}(f^{-1}(\mathrm{Cl}(V)))$. Therefore, we have $x \in f^{-1}(V) - \mathrm{mInt}(f^{-1}(\mathrm{Cl}(V))) \subset \bigcup_{G\in\sigma}\{f^{-1}(G) - \mathrm{mInt}(f^{-1}(\mathrm{Cl}(G)))\}$. Conversely, let $x \in \bigcup_{G\in\sigma}\{f^{-1}(G) - \mathrm{mInt}(f^{-1}(\mathrm{Cl}(G)))\}$. Then, there exists

$V \in \sigma$ such that $x \in f^{-1}(V) - \mathrm{mInt}(f^{-1}(\mathrm{Cl}(V)))$. By Theorem 1, we obtain $x \in D_{wmc}(f)$.

Similarly, by Theorems 2 and 3 we obtain the following theorems.

Theorem 5 *For a function $f : (X, m_X) \to (Y, \sigma)$, the following properties are hold:*

$$\begin{aligned} D_{wmc}(f) &= \bigcup_{B \in \mathcal{P}(Y)} \{\mathrm{mCl}(f^{-1}(\mathrm{Int}(\mathrm{Cl}_\theta(B)))) - f^{-1}(\mathrm{Cl}_\theta(B))\} \\ &= \bigcup_{B \in \mathcal{P}(Y)} \{\mathrm{mCl}(f^{-1}(\mathrm{Int}(\mathrm{Cl}(B)))) - f^{-1}(\mathrm{Cl}_\theta(B))\} \\ &= \bigcup_{G \in \sigma} \{\mathrm{mCl}(f^{-1}(\mathrm{Int}(\mathrm{Cl}(G)))) - f^{-1}(\mathrm{Cl}(G))\} \\ &= \bigcup_{K \in \mathrm{RC}(Y)} \{\mathrm{mCl}(f^{-1}(\mathrm{Int}(K))) - f^{-1}(K)\} \\ &= \bigcup_{G \in \beta(Y)} \{\mathrm{mCl}(f^{-1}(\mathrm{Int}(\mathrm{Cl}(G)))) - f^{-1}(\mathrm{Cl}(G))\} \\ &= \bigcup_{G \in \mathrm{SO}(Y)} \{\mathrm{mCl}(f^{-1}(\mathrm{Int}(\mathrm{Cl}(G)))) - f^{-1}(\mathrm{Cl}(G))\} \\ &= \bigcup_{A \in \mathcal{P}(X)} \{\mathrm{mCl}(A) - f^{-1}(\mathrm{Cl}_\theta(f(A)))\} \\ &= \bigcup_{B \in \mathcal{P}(Y)} \{\mathrm{mCl}(f^{-1}(B)) - f^{-1}(\mathrm{Cl}_\theta(B))\}, \end{aligned}$$

where $\mathrm{RC}(Y)$ *is the set of all regular closed sets of* Y.

Theorem 6 *For a function $f : (X, m_X) \to (Y, \sigma)$, the following properties are hold:*

$$\begin{aligned} D_{wmc}(f) &= \bigcup_{G \in \mathrm{PO}(Y)} \{\mathrm{mCl}(f^{-1}(\mathrm{Int}(\mathrm{Cl}(G)))) - f^{-1}(\mathrm{Cl}(G))\} \\ &= \bigcup_{G \in \mathrm{PO}(Y)} \{\mathrm{mCl}(f^{-1}(G)) - f^{-1}(\mathrm{Cl}(G))\} \\ &= \bigcup_{G \in \mathrm{PO}(Y)} \{f^{-1}(G) - \mathrm{mInt}(f^{-1}(\mathrm{Cl}(G)))\}. \end{aligned}$$

Remark 5 If $m_X = \tau$, SO(X), PO(X), or $\alpha(X)$, then we obtain the corresponding results from Theorems 4, 5 and 6. For example, in case $m_X = \mathrm{SO}(X)$ put $D_{wsc}(f) = \{x \in X : f$ is not weakly semi-continuous at $x\}$. Then we obtain the following three corollaries.

Corollary 1 *For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties hold:*

$$\begin{aligned} D_{wsc}(f) &= \bigcup_{G \in \sigma} \{f^{-1}(G) - \mathrm{sInt}(f^{-1}(\mathrm{Cl}(G)))\} \\ &= \bigcup_{H \in \mathcal{F}} \{\mathrm{sCl}(f^{-1}(\mathrm{Int}(H))) - f^{-1}(H)\} \\ &= \bigcup_{B \in \mathcal{P}(Y)} \{\mathrm{sCl}(f^{-1}(\mathrm{Int}(\mathrm{Cl}(B)))) - f^{-1}(\mathrm{Cl}(B))\} \\ &= \bigcup_{B \in \mathcal{P}(Y)} \{f^{-1}(\mathrm{Int}(B)) - \mathrm{sInt}(f^{-1}(\mathrm{Cl}(\mathrm{Int}(B))))\} \\ &= \bigcup_{G \in \sigma} \{\mathrm{sCl}(f^{-1}(G)) - f^{-1}(\mathrm{Cl}(G))\}, \end{aligned}$$

where $\mathcal{F}$ *is the family of closed sets of* (Y, σ).

Corollary 2 *For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties are hold:*

$$\begin{aligned} D_{wsc}(f) &= \bigcup_{B \in \mathcal{P}(Y)} \{\mathrm{sCl}(f^{-1}(\mathrm{Int}(\mathrm{Cl}_\theta(B)))) - f^{-1}(\mathrm{Cl}_\theta(B))\} \\ &= \bigcup_{B \in \mathcal{P}(Y)} \{\mathrm{sCl}(f^{-1}(\mathrm{Int}(\mathrm{Cl}(B)))) - f^{-1}(\mathrm{Cl}_\theta(B))\} \\ &= \bigcup_{G \in \sigma} \{\mathrm{sCl}(f^{-1}(\mathrm{Int}(\mathrm{Cl}(G)))) - f^{-1}(\mathrm{Cl}(G))\} \\ &= \bigcup_{K \in \mathrm{RC}(Y)} \{\mathrm{sCl}(f^{-1}(\mathrm{Int}(K))) - f^{-1}(K)\} \\ &= \bigcup_{G \in \beta(Y)} \{\mathrm{sCl}(f^{-1}(\mathrm{Int}(\mathrm{Cl}(G)))) - f^{-1}(\mathrm{Cl}(G))\} \\ &= \bigcup_{G \in \mathrm{SO}(Y)} \{\mathrm{sCl}(f^{-1}(\mathrm{Int}(\mathrm{Cl}(G)))) - f^{-1}(\mathrm{Cl}(G))\} \\ &= \bigcup_{A \in \mathcal{P}(X)} \{\mathrm{sCl}(A) - f^{-1}(\mathrm{Cl}_\theta(f(A)))\} \\ &= \bigcup_{B \in \mathcal{P}(Y)} \{\mathrm{sCl}(f^{-1}(B)) - f^{-1}(\mathrm{Cl}_\theta(B))\}, \end{aligned}$$

where $\mathrm{RC}(Y)$ *is the set of all regular closed sets of* Y.

Corollary 3 *For a function $f : (X, \tau) \to (Y, \sigma)$, the following properties are hold:*

$$\begin{aligned} D_{wsc}(f) &= \bigcup_{G \in \mathrm{PO}(Y)} \{\mathrm{sCl}(f^{-1}(\mathrm{Int}(\mathrm{Cl}(G)))) - f^{-1}(\mathrm{Cl}(G))\} \\ &= \bigcup_{G \in \mathrm{PO}(Y)} \{\mathrm{sCl}(f^{-1}(G)) - f^{-1}(\mathrm{Cl}(G))\} \\ &= \bigcup_{G \in \mathrm{PO}(Y)} \{f^{-1}(G) - \mathrm{sInt}(f^{-1}(\mathrm{Cl}(G)))\}. \end{aligned}$$

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Asupra punctelor de slab *m*-continuitate si slab *m*-discontinuitate

Rezumat

In aceasta lucrare vom defini noţiunea de functie slab m-continuă în un punct al domeniului, vom obţine unele caracterizări a acestor funcţii şi vom caracteriza mulţimea punctelor de slab m-discontinitate.