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Optical Properties of Vacuum Modelled as e-p Plasma

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Abstract

In this paper, using the electron-positron (e-p) plasma model for the physical vacuum, we study how to change the optical properties of vacuum in interaction with gravitational field.

Keywords: *e-p plasma, gravitational field, Boltzmann distribution, refractive index*

Introduction

In phenomenological theories of gravity, according to Wilson's - Dicke hypothesis [1-4] the gravitational interaction has an electromagnetic nature. The gravitational interaction is generated as a result of changing vacuum's properties after its interaction with charged particles. Neutral bodies are considered as systems consisting of particles with charge. According to these theories, static gravitational interaction is, like electrostatic interaction, an interaction where the Gauss's law is valid (field strength is proportional to $1/r^2$) and the deviations from this law (general relativistic effects) are the result of changes in optical properties of vacuum interaction with the substance [5].

In this paper we analyze the optical properties of vacuum plasma modeled as pairs of particle - antiparticle. This plasma, homogeneous and isotropic at large distances from bodies, becomes inhomogeneous and anisotropic in interaction with gravity field generated by these. In the first part of the work we establish the link between plasma parameters and optical parameters of the plasma. In part two the change of the plasma parameters is analyzed, implicitly, the optical parameters too, in the presence of a gravitational field.

Refractive Index of Plasma

In Stochastic Physics [5,6], the vacuum (the physical one) is modeled as: a background of electromagnetic waves with temperature $T = 0\text{K}$ (CZPF, Classical Zero Point Field - in Stochastic Electrodynamics, SED), as a pair plasma particle - antiparticle or as a system of neutral particles (in Stochastic Mechanics, MS).

Far from the substance, the physical vacuum has the structure of a mixture of particle – antiparticle pairs, both in the free state and in bounded state (neutral systems - atoms). As, according to theory [7], the probability of generating pairs is inversely proportional to their mass, the electron-positron pair (e-p) concentration is at least one hundred times higher than other pairs' with greater mass (i.e. mesons, protons etc.). For this reason, that plasma consists of electrons and positrons, free or bounded in positronium atoms.

Between e-p pairs plasma model and the model of CZPF background radiation, proposed by EDS for the physical vacuum, there exists a direct connection, in that the pair appears as fluctuations in this background [8].

Far from the substance, the CZPF is homogeneous and isotropic and, therefore, e-p plasma is homogeneous and isotropic as well, with the concentration of pairs (number of pairs per unit volume) N_0. If the fraction of free particles is f_0, then their concentration is $N_e = N_p = f_0 N_0$ and the concentration the positronium atoms is $N_a = \left(1 - f_0\right) N_0$.

According to electromagnetic theory [9] and the theory of light propagation [10], optical properties of a medium composed by atoms, molecules, ions and electrons, are characterized by refractive index which is a function of electrical and magnetic properties described by permittivity ε and permeability μ. The relation between the refractive index and relative permittivity and relative permeability is the following

$$n = \left(\varepsilon\mu\right)^{1/2} . \tag{1}$$

Relative permittivity is related to polarisability coefficient α

$$\alpha = \frac{q^2}{m_e \varepsilon_0} \sum \frac{N_j}{\omega_j^2 - \omega^2 - \omega^3 i\Gamma} = \frac{4\pi e^2}{m_e} \sum \frac{N_j}{D_j} \tag{2}$$

by the relation Clausius - Mosotti

$$\frac{\varepsilon - 1}{\varepsilon + 2} = \frac{N\alpha}{3} \quad \text{or} \quad \varepsilon = \frac{3 + 2N\alpha}{3 - N\alpha} , \tag{3}$$

where N is the concentration of the substance, N_i is the number of electrons with proper frecvency ω_j.

When substance $\mu = 1$, the equation (1) becomes

$$n = \varepsilon^{1/2} . \tag{4}$$

With this and the relation (2), we obtain the Lorentz - Lorentz formula for the refractive index of the substance

$$\frac{n^2 - 1}{n^2 + 2} = \frac{N\alpha}{3} \quad \text{or} \quad n^2 = \frac{3 + 2N\alpha}{3 - N\alpha} . \tag{5}$$

If the concentration N is low (gas), there results

$$n^2 = \varepsilon = 1 + N\alpha \quad \text{or} \quad n = 1 + N\frac{\alpha}{2} . \tag{6}$$

If $N = 0$, there results the refractive index of vacuum

$$n = \varepsilon = 1 .$$

If the gas contains free charged particles (it is partially ionized), the permittivity expression enters a specific component of these

$$\varepsilon = 1 + N_a\alpha - \sum_j \frac{\omega_{Pj}^2}{\omega^2}, \tag{7}$$

where ω_{Pj} is the plasma frequency for j component,

$$\omega_{Pj}^2 = \frac{q_j^2 N_j}{m_j \varepsilon_0}. \tag{8}$$

Similarly, the physical vacuum permittivity is determined both by neutral systems with concentration $N_a = N_0\left(1 - f_0\right)$ and the free electrons and positrons with concentration $N_e = N_p = N_0 f_0$, so the relative permittivity of the vacuum away from substance is

$$\varepsilon_{or} = N_0(1-f_0)\alpha - \frac{\omega_{pp}^2}{\omega^2} - \frac{\omega_{pe}^2}{\omega^2} = N_0(1-f_0)\frac{4\pi e_v^2}{m_{ev}}\frac{1}{D_v} - 2N_0 f_0 \frac{4\pi e_v^2}{m_{ev}\omega^2}, \tag{9}$$

where m_{ev} is the electron mass in vacuum (and positron), $e_v^2 = e^2/\varepsilon_v$ is the square of electric charge in vacuum, ε_v is the relative permittivity of the internal vacuum, ω_0 is fundamental frequency of the positron (atom) and

$$D_v = \omega_0^2 - \omega^2 - i\omega^3\Gamma. \tag{10}$$

If $f_0 << 1$ and $\omega << \omega_0$ pulsation and the relative permittivity is equal to unity, there results a first relation between the internal parameters of the physical vacuum

$$f_0 = 1 - \frac{m_{ev}\omega_0^2}{4\pi e_v^2 N_0} \quad \text{or} \quad \frac{m_{ev}\omega_0^2}{4\pi e_v^2 N_0} = 1. \tag{11}$$

The e-p Plasma Properties in the Gravitational Field

Consider a body with the mass M generating a static gravitational field with intensity

$$\Gamma(r) = -\frac{GM}{r^2}. \tag{12}$$

The e-p plasma components interact with this field, inducing a compression and thus the plasma concentration is dependent on the location and field source $N(M,r)$. In the following, we establish the expression of this dependence in the general case when compression is polytropic. We consider the plasma modified by the gravitational field as a gas with pressure p, temperature T and fraction of charged particles f. In this case, the concentration of free electrons and positrons is $N_e = N_p = fN$ and the concentration of positronium atoms is $N_a = \left(1 - f\right)N$.

For a plasma thermally excited, according to M. N. Saha relation [11, 12], between the temperature T, pressure p and the fraction f, there is the following relationship

$$\frac{f^2}{1-f^2}p = 2\frac{g_e}{g_a}\left(\frac{2\pi m_{ev}}{h^2}\right)^{3/2}\left(kT\right)^{5/2}\exp\left(-\frac{E_i}{kT}\right). \tag{13}$$

Total pressure p is given by

$$p = p_e + p_p + p_a = N(1+f)kT \quad . \tag{14}$$

Parameters g_e and g_a are the statistical weights of electrons and atoms, so

$$\frac{g_e}{g_a} = \exp\frac{-E_{p\infty}+E_{p0}}{kT} = \exp\left(-\frac{E_i}{kT}\right) \tag{15}$$

and $E_i = E_{p\infty} - E_{p0}$ is the ionization energy and, simultaneously, the binding energy of the positronium $E_i = \left(2m_{ev} - m_a\right)c^2$.

Replacing equation (15) in (13), there results

$$\frac{f^2}{1-f^2}p = 2\left(\frac{m_{ev}}{2\pi\hbar^2}\right)^{3/2}\left(kT\right)^{5/2}\exp\left(-\frac{2E_i}{kT}\right) . \tag{16}$$

Far from the body, $r \to \infty$, the vacuum is homogeneous and isotropic with concentration N_0, the equivalent temperature T_0 and degree of ionization f_0, so that Saha's equation (16) becomes

$$\frac{f_0^2}{1-f_0^2} = 2\left(\frac{m_{ev}}{2\pi\hbar^2}\right)^{3/2}\left(kT_0\right)^{5/2}\exp\left(-\frac{2E_i}{kT_0}\right) . \tag{17}$$

In the presence of a body with mass M, the fluid undergoes gravitational compression that leads to the change of parameters N, p, T and f that will become the function of the initial parameters: N_0, p_0, T_0, f_0, the mass M and the distance r between mass and the point where parameters are evaluated.

The dependency functions are determined for the general case of a polytropic compression with s exponent by solving the system of equations consisting of: Saha's equation (16), the static fundamental law of fluid corresponding to this situation:

$$dp = 2dp_e + dp_a = -\frac{GMNdr}{r^2}\left[m_a + f\left(2m_{ev} - m_a\right)\right] , \tag{18}$$

the law of polytropic process

$$pV^s = p_0V_0^s = \text{const.} \tag{19a}$$

or

$$T = T_0\left(\frac{p_0}{p}\right)^{\frac{1-s}{s}} \tag{19b}$$

and state law (14).

To determine the dependency relation for the pressure $p(r)$, we use the relations (14), (16), (18) and (19b).

First, by substituting (19b) in (16), there results

$$\frac{f^2}{1-f^2}p^{\frac{5-3s}{2s}}\exp\left[\left(\frac{2E_i}{kT_0}\right)\left(\frac{p_0}{p}\right)^{\frac{s-1}{s}}\right] = 2\left(\frac{m_{ev}}{2\pi\hbar^2}\right)^{\frac{3}{2}}\left(kT_0\right)^{5/2}p_0^{\frac{5(1-s)}{2s}} = A = \text{const.} \tag{20}$$

From equation (20) there results the expression of this fraction f

$$f=\left\{1+A^{-1}p^{\frac{5-3s}{2s}}\exp\left[\left(\frac{2E_i}{kT_0}\right)\left(\frac{p_0}{p}\right)^{\frac{s-1}{s}}\right]\right\}^{-\frac{1}{2}}. \tag{21}$$

From relations (14) and (19b), there results

$$N=\frac{p_0^{\frac{s-1}{s}}}{kT_0}\frac{p^{\frac{1}{s}}}{\left(1+f\right)}. \tag{22}$$

Replacing (21) in (22) and the expression obtained in (4), there result the differential equations with variables p and r

$$\frac{\left\{1+\left[1+A^{-1}p^{\frac{5-3s}{2s}}\exp\left(\left(\frac{2E_i}{kT_0}\right)\left(\frac{p_0}{p}\right)^{\frac{s-1}{s}}\right)\right]^{-\frac{1}{2}}\right\}p_0^{\frac{1-s}{s}}dp}{p^{\frac{1}{s}}\left\{1-\left(1-\frac{2m_{ev}}{m_a}\right)\left[1+A^{-1}p^{\frac{5-3s}{2s}}\exp\left(\left(\frac{2E_i}{kT_0}\right)\left(\frac{p_0}{p}\right)^{\frac{s-1}{s}}\right)\right]^{-\frac{1}{2}}\right\}}=-\frac{GMm_a}{kT_0}\frac{dr}{r^2}. \tag{23}$$

The left side of the differential equation is difficult to integrate. A solution can be obtained only when $f \ll 1$, valid for weak fields, $r \gg GM/c^2$. Imposing this condition, relation (21) can be approximated as follows

$$f_a\cong A^{\frac{1}{2}}p^{\frac{3s-5}{4s}}\exp\left[\left(\frac{-E_i}{kT_0}\right)\left(\frac{p_0}{p}\right)^{\frac{s-1}{s}}\right]. \tag{24}$$

Replacing (24) in (22), this results in (4) and, taking into account $1-2m_{ev}/m_a\cong -E_i/c^2$, there follows

$$\left[1+f_a\left(1-\frac{E_i}{2m_{ev}c^2}\right)\right]p_0^{\frac{1-s}{s}}p^{\frac{-1}{s}}dp=-\frac{GMm_a}{kT_0}\frac{dr}{r^2} \tag{25a}$$

or

$$\int_{p_0}^{p}\left[1+f_a\left(1-\frac{E_i}{2m_{ev}c^2}\right)\right]p_0^{\frac{1-s}{s}}p^{\frac{-1}{s}}dp=-\frac{GMm_a}{kT_0}\int_r^{\infty}\frac{dr}{r^2}. \tag{25b}$$

The integral on the left can be split in two:

$$I_1=p_0^{\frac{1-s}{s}}\int_p^{p_0}p^{\frac{-1}{s}}dp=\frac{s}{s-1}p_0^{\frac{1-s}{s}}\left(p_0^{\frac{1-s}{s}}-p^{\frac{1-s}{s}}\right) \tag{26}$$

and

$$I_2=\left(1-\frac{E_i}{2m_{ev}c^2}\right)A^{\frac{1}{2}}p_0^{\frac{1-s}{s}}\int_p^{p_0}p^{\frac{3s-9}{4s}}\exp\left[\left(-\frac{E_i}{kT_0}\right)\left(\frac{p_0}{p}\right)^{\frac{s-1}{s}}\right]dp. \tag{27}$$

In integral I_2, when approximating

$$\exp\left[\left(\frac{-E_i}{kT_0}\right)\left(\frac{p_0}{p}\right)\right]^{\frac{s-1}{s}} \cong 1-\left[\left(\frac{-E_i}{kT_0}\right)\left(\frac{p_0}{p}\right)\right]^{\frac{s-1}{s}}$$

there results

$$I_2 \cong \left(1-\frac{E_i}{2m_{ev}c^2}\right)A^{\frac{1}{2}}p_0^{\frac{1-s}{s}}\int_p^{p_0} p^{\frac{3s-9}{4s}}\left[1-\left(\frac{E_i}{kT_0}\right)\left(\frac{p_0}{p}\right)^{\frac{s-1}{s}}\right]dp \quad . \tag{28a}$$

Integrating, we obtain

$$I_2 \cong \left(1-\frac{E_i}{2m_{ev}c^2}\right)A^{\frac{1}{2}}p_0^{\frac{1-s}{s}}\left[\frac{4s}{7s-9}\left(p_0^{\frac{7s-9}{4s}}-p^{\frac{7s-9}{4s}}\right)-\left(\frac{E_i}{kT_0}\right)sp_0^{\frac{s-1}{s}}\left(p^{\frac{1}{s}}-p_0^{\frac{1}{s}}\right)\right] . \tag{28b}$$

From (25), integrating after r generates

$$I(r)=\frac{GMm_a}{kT_o}\int_r^{\infty}\frac{-dr}{r^2}=-\frac{GMm_a}{kT_or} . \tag{29a}$$

Replacing (26) and (28b) and (29a) in (25b), there results

$$\frac{s}{s-1}+A^{\frac{1}{2}}\left(1-\frac{E_i}{2m_{ev}c^2}\right)\left[\frac{4s}{7s-9}p_0^{\frac{3s-5}{4s}}+\left(\frac{E_i}{kT_0}\right)sp_0^{\frac{1}{s}}\right]-\frac{s}{s-1}p_0^{\frac{1-s}{s}}p^{\frac{s-1}{s}}-$$
$$A^{\frac{1}{2}}\left(1-\frac{E_i}{2m_{ev}c^2}\right)p_0^{\frac{1-s}{s}}\left[\frac{4s}{7s-9}p^{\frac{7s-9}{4s}}+\left(\frac{E_i}{kT_0}\right)sp_0^{\frac{s-1}{s}}p^{\frac{1}{s}}\right]=-\frac{GMm_a}{kT_or}. \tag{29b}$$

If one neglects the terms proportional with $A^{1/2}<<1$, there results

$$p=p_0\left(1+\frac{s-1}{s}\frac{GMm_a}{kT_or}\right)^{\frac{s}{s-1}} \cong p_0\left[1+\frac{GMm_a}{kT_or}+2\frac{s-1}{s}\left(\frac{GMm_a}{kT_or}\right)^2\right] . \tag{29c}$$

By replacing (29c) in expression (22) for concentration, with the approximation $1+f\cong 1$, there results a Boltzmann-type distribution for concentration

$$N=\frac{p_0}{kT_0}\left(1+\frac{s-1}{s}\frac{GMm_a}{kT_or}\right)^{\frac{1}{s-1}}=N_0\left(1+\frac{s-1}{s}\frac{GMm_a}{kT_or}\right)^{\frac{1}{s-1}} . \tag{30}$$

When replacing this expression of concentration in the expression of relative permittivity ε_r, for the vacuum modified as a result of its interaction with a body, there results a dependency of this, and implicitly, of the refractive index, by the mass M and distance r, similar to that obtained by means of other methods in the literature [2, 13, 14].

Conclusions

E-p plasma model allows obtaining the expression of gravitational refractive index. The e-p plasma compression is most likely of adiabatic type. In this case, the adiabatic exponent is $s=5/3$ and the concentration $N=N_0\left[1+\frac{2x}{5}\right]^{\frac{3}{2}}\cong N_0\left[1+\frac{3x}{5}\right]$, where $x=GMm_a/\left(kT_or\right)$.

From this relation of concentration, there may be obtained an expression of the gravitational index of refraction with a similar dependency to that deduced in other theories. The consequences of the model are the subject of a future work.

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Proprietățile optice ale vidului modelat ca plasmă e-p

Rezumat

În această lucrare, folosind modelul plasmă electron-pozitron (e-p) pentru vidul fizic, studiem cum se modifică proprietățile optice ale acestuia în interacțiune cu câmpul gravitațional.