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Well-Posedness of a Common Fixed Point Problem for Three Mappings under Strict Contractive Conditions

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Abstract

The purpose of this paper is to extend the notion of well-posednes of fixed point problem for a mapping to a set of mappings. Also, we extend the property (E.A) known for two mappings to the general case of sets of mappings. Using weak compatibility and these new concepts, we prove a general common fixed point theorem for three self-mappings satisfying a general implicit relation and using the extended property (E.A) for which the fixed point problem is well posed.

Key words: *well-posednes of fixed point problem for a set of mappings, common fixed points, implicit relations, extended property (E.A.), weakly compatible mappings*

Introduction

Let (X, d) be a metric space and S, T two self-mappings of X. In [4], Jungck defined S and T to be compatible if

$$\lim_{n\to\infty} d(STx_n, TSx_n) = 0$$

whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t$$

for some $t \in X$. This concept was frequently used to prove existence theorems in common fixed point theory. The study on common fixed point theory for noncompatible mappings is also interesting. Work along this lines has been recently initiated by Pant [7], [8], [9].

In [2], Aamri and Moutawakil introduced a generalization of the concept of noncompatible mappings.

Definition 1. Let S and T be two self-mappings of a metric space (X, d). We say that S and T satisfy property (E.A) if there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t$$

for some $t \in X$.

Remark 1. It is clear that two self-mappings of a metric space (*X*, *d*) will be noncompatible if there exists at lest one sequence $\{x_n\}$ in *X* such that

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t$$

for some $t \in X$, but

$$\lim_{n\to\infty} d(STx_n, TSx_n)$$

is either non-zero or not exists.

Therefore two noncompatible self-mappings of a metric space (*X*, *d*) satisfy property (E.A).

Definition 2 ([5]). Two self mappings of a metric space (*X*, *d*) are said to be weakly compatible if *Tu* = *Su* for some $u \in X$, then *STu* = *TSu.*

Two compatible mappings are weakly compatible.

Definition 3 ([3]). Let (*X*, *d*) be a metric space and $f : (X, d) \to (X, d)$ be a mapping. The fixed point problem for *f* is said to be well-posed if:

(i) f has a unique fixed point *x* in *X*,

(ii) for any sequence $\{x_n\}$ of points in *X* such that $\lim_{n\to\infty} d(Tx_n x_n) = 0$ we have $\lim_{n\to\infty} d(x_n x) = 0$.

Recently, the well-posedness of the fixed point problem for certain types of mappings has been investigated in [14], [6], [12], [13], [1] and other papers.

The notion of function satisfying an implicit relation is first introduced in [10], [11] and other papers.

The purpose of this paper is two fold. Firstly, we extend property (E.A) to the general case of an arbitray set mappings (see Definition 4). Secondly, we generalize the notion of well-posedness to the general case of an arbitrary set of mappings (see Definition 5). Using weak compatibility and these new concepts, we prove a general common fixed point theorem (see Theorem 1) for a set of three self-mappings satisfying the extended property (E.A). We show (see Theorem 4) that the common fixed point problem for these mappings is well-posed when the implicit relations satisfy some suitable conditions.

Implicit Relations

1. Let $F(t_1,...,t_6) : \boldsymbol{R}^6 \to \boldsymbol{R}$ be a continuous mapping. We define the following properties:

(F1): $F(t, 0, 0, t, t, 0) > 0$, for every $t > 0$.
(F2): $F(t, 0, t, 0, 0, t) > 0$, for every $t > 0$.
(F_u): $F(t, t, 0, 0, t, t) \geq 0$, for every $t > 0$.

Example 1.1. $F(t_1,...,t_6): t_1 - c\max\left\{t_2, \frac{t_3+t_4}{2}, \frac{t_5+t_6}{2}\right\}$, where $0 < c \le 1$.

(F1): $F(t,0,0,t,t,0) = t\left(1-\frac{c}{2}\right) > 0$, for every $t > 0$.

(F2): $F(t,0,t,0,0,t) = t\left(1-\frac{c}{2}\right) > 0$, for every $t > 0$.

(F_u): $F(t,t,0,0,t,t) = t(1-c) \ge 0$, for every $t > 0$.

Example 1.2. $F(t_1,...,t_6): t_1 - c\max\left\{\frac{t_2+t_3}{2}, \sqrt{t_4t_6}, \sqrt{t_5t_6}\right\}$, where $0 < c \le 1$.

(F1): $F(t,0,0,t,t,0) = t > 0$, for every $t > 0$.

(F2): $F(t,0,t,0,0,t) = t\left(1-\frac{c}{2}\right) > 0$, for every $t > 0$.

(F_u): $F(t,t,0,0,t,t) = t(1-c) \ge 0$, for every $t > 0$.

Example 1.3. $F(t_1,...,t_6): t_1^3 - at_1^2t_2 - bt_1t_3t_4 - ct_5^2t_6 - dt_5t_6^2$, where $a,b,c,d \ge 0$ and $0 < a+c+d \le 1$.

(F1): $F(t,0,0,t,t,0) = t^3 > 0$, for every $t > 0$.

(F2): $F(t,0,t,0,0,t) = t^3 > 0$, for every $t > 0$.

(F_u): $F(t,t,0,0,t,t) = t^3(1-(a+c+d)) \ge 0$, for every $t > 0$.

Example 1.4. $F(t_1,...,t_6): t_1^4 - c\frac{t_3^2t_4^2 + t_5^2t_6^2}{1+t_2+t_3+t_4}$, where $0 < c \le 1$.

(F1): $F(t,0,0,t,t,0) = t^4 > 0$, for every $t > 0$.

(F2): $F(t,0,t,0,0,t) = t^4 > 0$, for every $t > 0$.

(F_u): $F(t,t,0,0,t,t) = t^4\left(1-\frac{c}{1+t}\right) \ge 0$, for every $t > 0$.

Example 1.5. $F(t_1,...,t_6): t_1^6 - c\frac{t_3^3t_4^3 + t_1t_2t_5^2t_6^2}{1+(t_3+t_4)^6}$, where $0 < c \le 1$.

(F1): $F(t,0,0,t,t,0) = t^6 > 0$, for every $t > 0$.

(F2): $F(t,0,t,0,0,t) = t^6 > 0$, for every $t > 0$.

(F_u): $F(t,t,0,0,t,t) = t^6(1-c) \ge 0$, for every $t > 0$.

2. Let $F(t_1,...,t_6): \boldsymbol{R}^6 \to \boldsymbol{R}$ be a mapping satisfying the following property:

(F_p): There exists $p \in (0,1)$ such that for every $u,v,w \ge 0$,

$$F(u,v,0,w,u,v) \le 0 \Rightarrow u \le p\max\{v,w\}.$$

Example 2.1. $F(t_1,...,t_6): t_1 - c\max\left\{t_2, \frac{t_3+t_4}{2}, \frac{t_5+t_6}{2}\right\}$, where $0 < c < 1$.

(F_p): For all $u,v,w \ge 0$, we have

$$F(u,v,0,w,u,v) = u - c\max\left\{v, \frac{w}{2}, \frac{u+v}{2}\right\}.$$

Suppose that $F(u,v,0,w,u,v) \le 0$, with $u > 0$ and $u \ge \max\{v,w\}$. Then we have

$$\max\left\{v, \frac{w}{2}, \frac{u+v}{2}\right\} \le \max\left\{u, \frac{u}{2}, \frac{u+v}{2}\right\} = u.$$

Therefore we get $u(1-c) \le 0$, a contradiction. Hence $0 < u \le \max\{v,w\}$, which implies that

$$u \le c\max\left\{v, \frac{w}{2}, \frac{u+v}{2}\right\} \le c\max\{v,w\}.$$

Hence $u \le c\max\{v,w\}$. This inequality is also true if $u = 0$. We conclude that (F_p) is satisfied with $p = c \in (0,1)$.

Example 2.2. $F(t_1,...,t_6) : t_1 - c\max\left\{\frac{t_2+t_3}{2}, \sqrt{t_4t_6}, \sqrt{t_5t_6}\right\}$, where $0 < c < 1$.

(F_p): For all $u,v,w \ge 0$, we have

$$F(u,v,0,w,u,v) = u - c\max\left\{\frac{v}{2}, \sqrt{wv}, \sqrt{uv}\right\}.$$

Suppose that $F(u,v,0,w,u,v) \le 0$, with $u > 0$ and $u \ge \max\{v,w\}$. Then we have

$$\max\left\{\frac{v}{2}, \sqrt{wv}, \sqrt{uv}\right\} \le \max\{u, \sqrt{uu}, \sqrt{uu}\} = u.$$

Therefore we get $u(1-c) \le 0$, a contradiction. Hence $0 < u \le \max\{v,w\}$, which implies that

$$u \le \max\left\{\frac{v}{2}, \sqrt{wv}, \sqrt{uv}\right\} \le$$

$$\le c\max\left\{\max\{v,w\}, \sqrt{\max\{v,w\}\max\{v,w\}}, \sqrt{\max\{v,w\}\max\{v,w\}}\right\} =$$

$$= c\max\{v, w\}.$$

Hence $u \le c\max\{v,w\}$. This inequality is also true if $u = 0$. We conclude that (F_p) is satisfied with $p = c \in (0,1)$.

Example 2.3. $F(t_1,...,t_6) : t_1^3 - at_1^2t_2 - bt_1t_3t_4 - ct_5^2t_6 - dt_5t_6^2$, where $a,b,c,d \ge 0$ and $0 < a+c+d < 1$.

(F_p): For all $u,v,w \ge 0$, we have

$$F(u,v,0,w,u,v) = u^3 - au^2v - cu^2v - duv^2.$$

Suppose that $F(u,v,0,w,u,v) \le 0$. If u > 0, then $u^2 \le auv + cuv + dv^2$. If $u \ge v$, then $u^2 \le (a+c+d)u^2 < u^2$, a contradiction. Hence $u < v$ and then we have $u^2 \le (a+c+d)v^2$, which implies $u \le pv \le p\max\{v,w\}$, where $0 < p := \sqrt{a+c+d} < 1$. If u = 0, then evidently, $u \le p\max\{v,w\}$. Thus (F_p) is satisfied.

Example 2.4. $F(t_1,...,t_6): t_1^4 - c\dfrac{t_3^2t_4^2 + t_5^2t_6^2}{1+t_2+t_3+t_4}$, where $0 < c < 1$.

(F_p): For all $u,v,w \ge 0$, we have

$$F(u,v,0,w,u,v) = u^4 - c\frac{u^2v^2}{1+v+w}.$$

Suppose that $F(u,v,0,w,u,v) \le 0$, then $u^4 \le c\dfrac{u^2v^2}{1+v+w}$, which implies $u^4 \le cu^2v^2$. If u > 0, then $u \le \sqrt{c}v \le \sqrt{c}\max\{v,w\}$. Hence we have $u \le p\max\{v,w\}$, where $0 < p := \sqrt{c} < 1$. If u = 0, then $u \le p\max\{v,w\}$. Thus the property (F_p) is satisfied.

Example 2.5. $F(t_1,...,t_6): t_1^6 - c\dfrac{t_3^3t_4^3 + t_1t_2t_5^2t_6^2}{1+(t_3+t_4)^6}$, where $0 < c < 1$.

(F_p): For all $u,v,w \ge 0$, we have

$$F(u,v,0,w,u,v) = u^6 - c\frac{u^3v^3}{1+w^6}.$$

Suppose that $F(u,v,0,w,u,v) \le 0$, then $u^6 \le c\dfrac{u^3v^3}{1+w^6}$, which implies $u^6 \le cu^3v^3$. If u > 0, then $u \le c^{\frac{1}{3}}v \le c^{\frac{1}{3}}\max\{v,w\}$. Hence we have $u \le p\max\{v,w\}$, where $0 < p := c^{\frac{1}{3}} < 1$. If u = 0, then $u \le p\max\{v,w\}$. Thus the property (F_p) is satisfied.

Common Fixed Points

Theorem 1. Let (X, d) be a metric space and $A,B,I:(X,d) \to (X,d)$ be three mappings satisfying the following inequality

$$F(d(Ax,By), d(Ix,Iy), d(Ix,Ax), d(Iy,By), d(Ix,By), d(Iy,Ax)) < 0, \quad (1)$$

for all $x,y \in X$ such that $x \ne y$, where F satisfies property (F_u). Then the mappings A, B and I have at most one common fixed point.

Proof. Suppose that A, B and I have two fixed points u, v with $u \ne v$. Then by (1) we have

$$F(d(Au,Bv), d(Iu,Iv), d(Iu,Au), d(Iv,Bv), d(Iu,Bv), d(Iv,Au)) =$$

$$= F(d(u,v), d(u,v), 0, 0, d(u,v), d(v,u)) < 0,$$

A contradiction of (F_u). □

To state our first main result, we need to introduce the following definition which extends Definition 1.

Definition 4. Let (X, d) be a metric space and $\mathcal{A}$ a set of self-mappings of X. We say that the mappings $A \in \mathcal{A}$ satisfy property (E.A) if there exists a sequence $\{x_n\}$ and some t in X such that

$$\lim_{n\to\infty} Ax_n = t, \ \forall A \in \mathcal{A}.$$

Our first main result runs as follows.

Theorem 2. Let A, B and I be three self-mappings of a metric space (X, d) such that:

(i) The pairs $\{A, I\}$ and $\{B, I\}$ are weakly compatible.
(ii) The mappings A, B and I satisfy the property (E.A).
(iii) The inequality (1) holds for all $x \neq y$ in X, where F satisfies properties (F_1), (F_2) and (F_u).
(iv) $I(X)$ is closed.

Then the mappings A, B and I have a unique common fixed point.

Proof. Since the set of mappings $\{A, B, I\}$ satisfies the property (E.A), there exists a sequence $\{x_n\}$ such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Ix_n = u, \tag{2}$$

for some $u \in X$.
Since $I(X)$ is closed there exists a point $a \in X$ such that $u = Ia$.
If the sequence $\{x_n\}$ satisfies

$$x_n = a, \ \forall n \geq n_0$$

for some positive integer n_0, then, from (2), we have

$$u = Aa = Ia = Ba.$$

So, we may suppose that $x_n \neq a$ for all integers n (otherwise, we consider a subsequence satisfying this property). In this case we use the inequality (1) and we obtain

$$F(d(Ax_n, Ba), d(Ix_n, Ia), d(Ix_n, Ax_n), d(Ia, Ba), d(Ix_n, Ba), d(Ia, Ax_n)) < 0.$$

Letting n tend to infinity we obtain:

$$F(d(Ia, Ba), 0, 0, d(Ia, Ba), d(Ia, Ba), 0) \leq 0,$$

which, by virtue of (F_1), implies that $d(Ia, Ba) = 0$. Hence $Ia = Ba$.
Since $x_n \neq a$ for all integers n, then by applying (2), we get

$$F(d(Aa, Bx_n), d(Ia, Ix_n), d(Ia, Aa), d(Ix_n, Bx_n), d(Ia, Bx_n), d(Ix_n, Aa)) < 0.$$

Letting n tend to infinity, we get

$$F(d(Aa, Ia), 0, d(Ia, Aa), 0, 0, d(Ia, Aa),) \leq 0,$$

which, by virtue of (F_2), implies that $d(Aa, Ia) = 0$. Hence $Aa = Ia$. Therefore, we obtain

$$Aa = Ia = Ba.$$

We set $x = Aa = Ia = Ba$. We shall prove that x is a common fixed point of the mappings A, B and I.
Since the pairs $\{A, I\}$ and $\{B, I\}$ are weakly compatible, then we have $AIa = IAa$ and $BIa = IBa$. Therefore

$$IIa = IAa = AIa = AAa = Ix = Ax, \tag{3}$$

and

$$IIa = IBa = BIa = BBa = Ix = Bx. \tag{4}$$

If $x = a$, then we have $x = Ax = Ix = Bx$. Therefore x is a common fixed point of the mappings A, B and I. So, we may suppose that $x \neq a$. In this case, by using the equalities (3) and (4) and the inequality (1), we have:

$$F(d(Aa,Bx),d(Ia,Ix),d(Ia,Aa),d(Ix,Bx),d(Ia,Bx),d(Ix,Aa)) =$$

$$= F(d(x,Ix),d(x,Ix),0,0,d(x,Ix),d(Ix,x)) < 0$$

a contradiction of (F_u) if $d(x, Ix) > 0$. Hence $Ix = x$. By virtue of (3) and (4), we conclude that

$$x = Ix = Ax = Bx.$$

Therefore x is a common fixed point of A, B and I.
By Theorem 1, the point x is the unique common fixed point of A, B and I. This completes the proof. □

By the lines of proof of Theorem 1, we obtain also the following result.

Theorem 3. Let A, B and I be three self-mappings of a metric space (X, d) such that:

(i) The pairs $\{A, I\}$ and $\{B, I\}$ are weakly compatible.
(ii) The mappings A, B and I satisfy the property (E.A).
(iii) The inequality (1) holds for all $x \neq y$ in X, where F satisfies properties (F_1), (F_2) and (F_u).
(iv) $A(X) \cup B(X) \subset I(X)$.

If one of $A(X)$, $B(X)$ or $I(X)$ is a closed subspace of X, then A, B and I have a unique common fixed point.

By our study we obtain a generalization of the Theorem 1 of [2]. More precisely, we have

Corollary 1. Let A, B and S be three self-mappings of a metric space (X, d) such that:

(i) The set of mappings $\{A, B, S\}$ satisfies property (E.A).
(ii) The pairs $\{A, S\}$ and $\{B, S\}$ are weakly compatible.
(iii) The inequality

$$d(Ax,Bx) < \max\left\{d(Sx,Sy),\frac{1}{2}(d(Sx,Ax)+d(Sy,By)),\frac{1}{2}(d(Sx,By)+d(Sy,Ax))\right\},$$

holds for all $x \neq y$ in X.
(iv) $A(X) \cup B(X) \subset S(X)$.

If one of $A(X)$, $B(X)$ or $S(X)$ is a complete subspace of X, then A, B and S have a unique common fixed point.

Proof. If Y is a subspace of the metric space (X, d) and Y is complete with the subspace metric, then Y is a closed subspace of X. Therefore we have that one of $A(X)$, $B(X)$ or $S(X)$ is closed and the proof follows from Theorem 2 and Example 1.1. □

Well-Posedness of Common Fixed Point Problem

We begin with the following definition which extends Definition 2.

Definition 5. Let (X, d) be a metric space and A a set of self-mappings of X. The common fixed point problem of the set A is said to be well-posed if:

(i) A has a unique common fixed point x in X. (That is, x is the unique point in X such that $Ax = x$ for all $A \in \mathcal{A}$).

(ii) For every sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} d(x_n, Ax_n) = 0, \ \forall A \in \mathcal{A},$$

we have

$$\lim_{n\to\infty} d(x_n, x) = 0.$$

Our second main result is concerned with the well-posedness of the common fixed point problem fot three mappings satisfying stricte contractive conditions. We have the following theorem.

Theorem 4. Let A, B and I be three self-mappings of a metric space (X, d) such that:

(i) The pairs $\{A, I\}$ and $\{B, I\}$ are weakly compatible.

(ii) The mappings A, B and I satisfy the property (E.A).

(iii) The inequality

$$F(d(Ax, By), d(Ix, Iy), d(Ix, Ax), d(Iy, By), d(Ix, By), d(Iy, Ax)) < 0, \tag{5}$$

holds for all $x \neq y$ in X, where F satisfies properties (F_1), (F_2) (F_u) and (F_p).

(iv) $I(X)$ is closed.

Then the common fixed point problem of A, B and I is well-posed.

Proof. By Theorem 2, the mappings A, B and I have a unique common fixed point x in X. Let $\{x_n\}$ be a sequence in X such that

$$\lim_{n\to\infty} d(x_n, Ax_n) = \lim_{n\to\infty} d(x_n, Bx_n) = \lim_{n\to\infty} d(x_n, Ix_n) = 0.$$

Without loss of generality, we may suppose that $x \neq x_n$ for every non-negative integer n. Then by inequality (5) we have

$$F(d(Ax, Bx_n), d(Ix, Ix_n), d(Ix, Ax), d(Ix_n, Bx_n), d(Ix, Bx_n), d(Ix_n, Ax)) =$$

$$= F(d(x, Bx_n), d(x, Ix_n), 0, d(Ix_n, Bx_n), d(x, Bx_n), d(Ix_n, x)) < 0.$$

Then, by property (F_p), we have

$$d(x, Bx_n) \leq p \max\{d(x, Ix_n), d(Ix_n, Bx_n)\} \leq p[d(x, Ix_n) + d(Ix_n, Bx_n)].$$

Therefore

$$d(x, x_n) \leq d(x, Bx_n) + d(Bx_n, x_n) \leq$$

$$\leq p[d(x, x_n) + d(x_n, Ix_n) + d(Ix_n, x_n) + d(x_n, Bx_n)] + d(Bx_n, x_n),$$

which implies

$$d(x, x_n) \le \frac{2p}{1-p} d(Ix_n, x_n) + \frac{p+1}{1-p} d(Bx_n, x_n) \to 0 \text{ as } n \to \infty .$$

Hence $\lim_{n\to\infty} d(x, x_n) = 0$. Consequently the common fixed point problem of the three mappings *A*, *B* and *I* is well posed. □

As a consequence, we have the following result.

Corollary 2. Let *A*, *B* and *S* be three self-mappings of a metric space (*X*, *d*) such that:

(i) The set of mappings {*A*, *B*, *S*} satisfies property (E.A).
(ii) The pairs {*A*, *S*} and {*B*, *S*} are weakly compatible.
(iii) The inequality

$$d(Ax, By) < c \max\left\{ d(Sx, Sy), \frac{1}{2}(d(Sx, Ax) + d(Sy, By)), \frac{1}{2}(d(Sx, By) + d(Sy, Ax)) \right\},$$

holds for all $x \ne y$ in *X*, where $c \in (0,1)$.
(iv) $A(X) \cup B(X) \subset S(X)$.

If one of *A*(*X*), *B*(*X*) or *S*(*X*) is a complete subspace of *X*, then the common fixed point problem of *A*, *B* and *S* is well posed.

Proof. The proof follows from Theorem 4 and Example 2.1. □

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O bună punere a unei probleme de punct fix pentru trei aplicații care satisfac o condiție de strictă contractivitate

Scopul acestei lucrări este să extindă noțiunea de bună punere a problemei de punct fix pentru o aplicație la o mulțime de aplicații. De asemenea, extindem proprietatea (E.A.) cunoscută pentru două aplicații la cazul general al mulțimilor de aplicații. Folosind noțiunea de compatibilitate slabă şi aceste noi concepte, demonstrăm o teoremă generală de punct fix comun pentru trei auto-aplicații care satisfac o relație implicită generală şi folosind proprietatea extinsă (E.A.) pentru care problema de punct fix este bine pusă.