BULETINUL Universităţii Petrol – Gaze din Ploieşti	Vol. LXII No. 1/2010	19 - 25	Seria Matematică - Informatică - Fizică

On Quasi-Irresolute Functions in Fuzzy Minimal Structures

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Abstract

The present paper attempts to generalize, for a fuzzy minimal structure, the concept of quasi-irresolute function introduced in the General Topology by V. Popa and T. Noiri. Several theorems are to be outlined, both referential theorems and theorems establishing equivalences between different concepts.

Keywords: *fuzzy topological space, fuzzy minimal structure, fuzzy Fm - quasi- irresolute function, fuzzy Fm - irresolute function, fuzzy Fm - contra-continuous function, fuzzy Fm - almost- contra-continuous function*

Preliminaries

Consider X an arbitrary non-empty set and the unit interval $J=[0,1]\subset \mathbf{R}$. A fuzzy set in X is an application $\lambda : X\to [0,1]$. $\mathcal{F}(X)$ marks the class of fuzzy sets in X. The X set, known as the X space, will be identified with the constant function **1** and the empty set Ø will be identified with the constant function **0**.

Consider I an index set and $\{\lambda_i\}_{i\in X}$ a class of fuzzy sets in X. The union and the intersection of this class, denoted as $\underset{i\in I}{\cup}\lambda_i$ and as $\underset{i\in I}{\cap}\lambda_i$ are defined by:

$$\underset{i\in I}{\cup}\lambda_i(x)=\sup_{i\in I}\lambda_i(x),\ \underset{i\in I}{\cap}\lambda_i(x)=\inf_{i\in I}\lambda_i(x),(\forall)x\in X.$$

If $\lambda_1,\lambda_2\in \mathcal{F}(X)$, the inclusion denoted as $\lambda_1\le\lambda_2$ or $\lambda_1\subseteq\lambda_2$, is defined by $\lambda_1(x)\le\lambda_2(x),(\forall)x\in X$ and the equality denoted as $\lambda_1=\lambda_2$ is defined by $\lambda_1(x)=\lambda_2(x),(\forall)x\in X$. Obviously, $\lambda_1=\lambda_2$ if and only if $\lambda_1\le\lambda_2$ and $\lambda_2\le\lambda_1$. The complement of $\lambda\in \mathcal{F}(X)$, denoted as λ^c, is defined by $\lambda^c=1-\lambda$, $\lambda^c(x)=1-\lambda(x),\ (\forall)x\in X$.

Consider X and Y two arbitrary non-empty sets, an application $f: X\to Y$ and $\lambda\in \mathcal{F}(X)$, $\mu\in\mathcal{F}(Y)$.

The image of λ is the fuzzy set $f(\lambda)\in\mathcal{F}(Y)$ defined by

$$f(\lambda)(y)=\begin{cases}\sup\limits_{x\in f^{-1}(y)} p\lambda(x), \text{ if } f^{-1}(y) \text{ is a nonempty set}\\ 0 \qquad\qquad\qquad , \text{otherwise}\end{cases}.$$

The inverse or reciprocal image of μ is the fuzzy set $f^{-1}(\mu)$ ∈ $\mathcal{F}(X)$ described by $f^{-1}(\mu)(x)=\mu(f_{(x)}), \forall x\in X$, that is $f^{-1}(\mu)=\mu\circ f$, in the sense of an ordinary composition of functions ([7]). The properties of f and f^{-1} are explained in ([13]).

A fuzzy point x_α in X is the fuzzy set in X which has the value α in the point $x\in X$ $(0<\alpha\leq 1)$ and the value 0 in the other points of the X space; we say that x_α has the support x (denoted as supp x_α=x) and the value α ([9]).We can write this

$$x_\alpha(y)=\begin{cases}\alpha, \ y=x\\ 0, \ y\neq x\end{cases}, \ y\in X .$$

A fuzzy set is equivalent with the union of all its fuzzy points. We assume that the fuzzy point x_α is an element of the fuzzy set $\lambda\in\mathcal{F}(X)$ and it is denoted as $x_\alpha\in\lambda$ if $\alpha\leq\lambda(x),(\forall)x\in X$. The relation $x_\alpha\in\bigcup_{i\in I}\lambda_i$ holds if there exists $i_0\in I$ so that $x_\alpha\in\lambda_{i_0}$. If x_α is a fuzzy point in X and f:$X\rightarrow Y$, then $f(x_\alpha)$ is a fuzzy point in Y; if supp x_α=x, then supp ($f(x_\alpha)$)= $f(x)$. If y_β is a fuzzy point in Y, then $f^{-1}(y_\beta)$ is a fuzzy point in X if $y_\beta\in f(x)$ and f is an injective function. In this case, if supp $y_\beta=y$, then supp ($f^{-1}(y_\beta)$)= $f^{-1}(y_\beta)$ ([13]).

We assume that the fuzzy point x_α is quasi-coincident (or q-coincident) with the set λ if $\alpha+\lambda(x)>1$, $x\in X$ and this is denoted as $x_\alpha q\lambda$; otherwise, we obtain $\alpha+\lambda(x)\leq 1$, which is as $x_\alpha\overline{q}\lambda$ ([9]).

The sets $\lambda,\mu\in\mathcal{F}(X)$ are said to be quasi-coincident sets (or q-coincident) if there exists $x\in X$ so that $\lambda(x)+\mu(x)>1$ and this is as $\lambda q\mu$; otherwise, we obtain $\lambda(x)+\mu(x)\leq 1$, which is denoted as $\lambda\overline{q}\mu$ ([9]). If λ and μ are q-coincident in X, then $\lambda(x)\neq 0$, $\mu(x)\neq 0$ and consequently $(\lambda\cap\mu)(x)\neq 0$ ([9]).

A fuzzy topology on X (according to Chang, [7]) is a class $\tau\leq\mathcal{F}(X)$ which satisfies the following conditions (or axioms):

(T_1) $\mathbf{0},\mathbf{1}\in\tau$;

(T_2) if $\delta_i\in\tau, i=\overline{1,n}$, then $\bigcap_{i=1}^{n}\delta_i\in\tau$, where $\bigcap_{i=1}^{n}\delta_i=\min_{1\leq i\leq n}\delta_i$;

(T_3) if $\delta_i\in\tau, i\in I$, then $\bigcup_{i\in I}\delta_i\in\tau$.

The couple (X,τ) is defined as a fuzzy topological space (according to Chang, [7]), abbreviated f.t.s. Each element of the τ class is a fuzzy open - τ set and the complement of a fuzzy open - τ set is called a fuzzy closed - τ set.

The interior and the closure of the set $\lambda\in\mathcal{F}(X)$ are defined by (see [7]):

$$\text{Int}\lambda=\overset{\circ}{\lambda}=\cup\{\delta;\ \delta\leq\lambda,\delta\in\tau\}, \ \text{Cl}\lambda=\overline{\lambda}=\cap\{\sigma;\ \sigma\geq\lambda,\sigma^c\in\tau\}.$$

Consider a f.t.s. (X,τ) and $\lambda\in\mathcal{F}(X)$. The set λ is called:

a) semi-open F, if $\lambda\leq\overline{\overset{\circ}{\lambda}}$;

b) regular closed F (resp. regular open F), if $\lambda=\overline{\overset{\circ}{\lambda}}$ (resp. $\lambda=\overset{\circ}{\overline{\lambda}}$) ([2]).

The complement of a semi-open F set is called semi-closed F set. The reunion of all semi-open F sets of the space X included in $\lambda \in \mathcal{F}(X)$ is called the semi-interior F of λ and is denoted as Fd Intλ or Fd $\mathring{\lambda}$. The intersection of all semi-closed F sets of the space X containing λ is called the F semi-closure of λ and it is denoted as FdCl or Fd $\overline{\lambda}$.

We assume that the fuzzy point x_α in X is a θ-semi-cluster point for $\lambda \in \mathcal{F}(X)$ if λq$\overline{\mu}$ for any μF semi-open set with x_αqμ. The set of all θ-semi-cluster points for λ is called the Fθ-semi-closure of λ and it is marked as Fθ-dC$l\overline{\lambda}$ or Fθ-d$\overline{\lambda}$. We say that the λ set is Fθ-semi-closed if λ= Fθ-d$\overline{\lambda}$. The complement of a Fθ-semi-closed set is called Fθ-semi-open ([5]).

Fuzzy Minimal Structures

V. Popa and T. Noiri have created and developed in [10] and [11] an extremely interesting unified theory of the main patterns of continuity, grounded on the concept of m-structure (or minimal structure), introduced by the same authors. As a generalization of the fuzzy domain, we have introduced the concept of fuzzy minimal structure (or F_m–structure) in [4].

We are going to review some of the definitions and some of the lemmas and theorems in [4].

Definition 1. Consider $\mathcal{F}(X)$ the class of the fuzzy subsets of the X space. We assume that the subclass $\mathcal{F}m_X \leq \mathcal{F}(X)$ is a fuzzy minimal structure on X (or a F_m-structure) if $\mathbf{0} \in \mathcal{F}m_X$, $\mathbf{1} \in \mathcal{F}m_X$. The couple $(X, \mathcal{F}m_X)$ is by definition a fuzzy minimal space or a F_m-space. The set $\lambda \in \mathcal{F}(X)$ is called F_m-open set if $\lambda \in \mathcal{F}m_X$; if $\lambda^c \in \mathcal{F}m_X$, then λ is called a F_m-closed set.

Remark 1. The definition of the fuzzy minimal structure maintains only the first condition of the definition of a fuzzy topology (according to Chang).

Definition 2. Consider $X \neq \emptyset$, $\mathcal{F}m_X$ a F_m-structure on X and $\lambda \in \mathcal{F}(X)$. Then, the F_m-closure and the F_m-interior are defined as:

a) $^{F}m_X - \overline{\lambda} = \cap \{\sigma; \lambda \leq \sigma, \sigma^c \in \mathcal{F}m_X\} = \inf\{\sigma; \lambda \leq \sigma, \sigma^c \in \mathcal{F}m_X\}$, respectively

b) $^{F}m_X - \mathring{\lambda} = \cup \{\delta; \delta \leq \lambda, \delta \in \mathcal{F}m_X\} = \sup\{\delta; \delta \leq \lambda, \delta \in \mathcal{F}m_X\}$.

The F_m-closure and the F_m-interior of λ are denoted as $\mathcal{F}m_X -$ Clλ, respectively $\mathcal{F}m_X -$Intλ.

Lemma 1. Consider $X \neq \emptyset$, $\mathcal{F}m_X$ a F_m-structure on X and $\lambda, \mu \in \mathcal{F}(X)$. Then the following propositions are true:

(1) $\mathcal{F}m_X - \overline{\lambda}^{\,c} = (m_X - \mathring{\lambda})^c$, $\mathcal{F}m_X - \mathring{\lambda}^{\,c} = (\mathcal{F}m_X - \overline{\lambda})^c$;

(2) If $\lambda^c \in \mathcal{F}m_X$ then $\mathcal{F}m_X - \overline{\lambda} = \lambda$, and if $\lambda \in \mathcal{F}m_X$ then $\mathcal{F}m_X - \mathring{\lambda} = \lambda$;

(3) $\mathcal{F}m_X - \overline{\mathbf{0}} = \mathbf{0}$, $\mathcal{F}m_X - \overline{\mathbf{1}} = \mathbf{1}$, $\mathcal{F}m_X - \mathring{\mathbf{1}} = \mathbf{1}$;

(4) If $\lambda \leq \mu$ then $\mathcal{F}m_X - \overline{\lambda} \leq \mathcal{F}m_X - \overline{\mu}$, $\mathcal{F}m_X - \mathring{\lambda} \leq \mathcal{F}m_X - \mathring{\mu}$;

(5) $\lambda \leq \mathcal{F}m_X - \overline{\lambda}$, $\mathcal{F}m_X - \mathring{\lambda} \leq \lambda$;

(6) $\mathcal{F}m_X - (\mathcal{F}m_X - \overline{\lambda}) = \mathcal{F}m_X - \overline{\lambda}$, $^{F}m_X -$Int $(\mathcal{F}m_X - \mathring{\lambda}) = \mathcal{F}m_X - \mathring{\lambda}$.

Lemma 2. Consider $(X, \mathcal{F}m_X)$ a F_m-space, $\lambda \in \mathcal{F}(X)$ and x_α a fuzzy point in X. Then $x_\alpha \in \mathcal{F}m_X - \overline{\lambda}$ if and only if μqλ for any set $\mu \in m_X$ satisfying the condition x_αqμ.

Definition 3. We say that $\mathcal{F}m_X$ has the property (***B***) if $(\delta_i)_{i\in I} \le \mathcal{F}m_X$ implies that $\bigcup_{i\in I}\delta_i \in \mathcal{F}m_X$.

Remark 2.This definition maintains only the conditions (T_1) and (T_3) of the definition of a fuzzy topology (according to Chang).

Remark 3. For a F_m-structure having the property (***B***), there can be used the fuzzy supra-topology terminology (see [8]). In this case the elements of the supra-topology are called fuzzy supra-open sets, and their complements are called fuzzy supra-closed sets (see [8]). We can define the supra-closure and the supra-interior of a fuzzy set by analogy with the already known definitions.

Lemma 3. Consider $X \ne \emptyset$, $\mathcal{F}m_X$ a fuzzy supra-topology on X and $\lambda \in \mathcal{F}(X)$. Then:

(1) $\lambda \in \mathcal{F}m_X$ if and only if $\mathcal{F}m_X - \lambda^\circ = \lambda$;

(2) λ is $\mathcal{F}m_X$ -closed if and only if $\mathcal{F}m_X$ - $\overline{\lambda} = \lambda$;

(3) $\mathcal{F}m_X - \lambda^\circ \in \mathcal{F}m_X$ and $\mathcal{F}m_X$ - $\overline{\lambda}$ is $\mathcal{F}m_X$ - closed.

Theorem 1. For a fuzzy minimal structure $\mathcal{F}m_X$ the following propositions are equivalent:

(1) $\mathcal{F}m_X$ is a supra-topology on X;
(2) If $\mathcal{F}m_X - \lambda^\circ = \lambda$, then $\lambda \in \mathcal{F}m_X$;
(3) If $\mathcal{F}m_X$ - $\overline{\mu} = \mu$, then $\mu^c \in \mathcal{F}m_X$.

Definition 4. Consider the fuzzy minimal space $(X, \mathcal{F}m_X)$ and the fuzzy topological space (Y, t). We say that $f:(X, \mathcal{F}m_X) \to (Y, t)$ is F_m-continuous (fuzzy) if for any fuzzy point x_α in X and for any set $\nu \in t$ with $f(x_\alpha)\mathrm{q}\nu$, there exists $\delta \in \mathcal{F}m_X$ with $x_\alpha \mathrm{q}\delta$ so that $f(\delta) \le \nu$. The f function is F_m-continuous (fuzzy) on X if it has this property in all the fuzzy points from X.

The next theorem is a characterization theorem for the F_m- continuous functions (fuzzy).

Theorem 2. Consider the fuzzy minimal space $(X, \mathcal{F}m_X)$, the fuzzy topological space (Y,t) and the function $f:(X, \mathcal{F}m_X) \to (Y, t)$. Then the following assertions are equivalent:

(1) f is F_m-continuous (fuzzy);
(2) $f^{-1}(\nu) = \mathcal{F}m_X$ - Int $f^{-1}(\nu)$, $(\forall)$ $\nu \in t$;
(3) $\mathcal{F}m_X$- $f^{-1}(\sigma) = f^{-1}(\sigma)$ $(\forall)$ $\sigma \in \mathcal{F}(Y)$ where $\sigma^c \in t$;
(4) $\mathcal{F}m_X$- $f^{-1}(\mu) \le f^{-1}(\overline{\mu})$, $(\forall)$ $\mu \in \mathcal{F}(Y)$;
(5) $f(\mathcal{F}m_X$ - $\overline{\lambda}) \le \overline{f(\lambda)}$, $(\forall)$ $\lambda \in \mathcal{F}(X)$;
(6) $f^{-1}(\mathring{\mu}) \le \mathcal{F}m_X$ - Int$f^{-1}(\mu)$, $(\forall)$ $\mu \in \mathcal{F}(Y)$.

Remark 4. Along the whole length of the paper, in the case of fuzzy spaces the common points of the space X are substituted by fuzzy points and there is used the q-coincident relation.

F_m-Quasi-Irresolute Functions

The fuzzy irresolute functions and the fuzzy quasi-irresolute functions have been introduced and studied in [3]. Further on, the concept of F_m-quasi-irresolute function has been introduced in [6], as follows.

Definition 5. Consider the fuzzy minimal space $(X, \mathcal{F}m_X)$, the fuzzy topological space (Y, t) and the function $f:(X, \mathcal{F}m_X) \to (Y, t)$. We say that the function f is F_m-quasi-irresolute in the fuzzy point x_α in X if for any set ν, F-semi-open in (Y, t) with $f(x_\alpha)\mathrm{q}\nu$, there exists $\delta \in \mathcal{F}m_X$ with $x_\alpha\mathrm{q}\delta$ so that $f(\delta) \le \overline{\nu}$. We say that the function f is F_m-quasi-irresolute on X if it has this property in all the fuzzy points in X.

As in [6] we have not focused too much on this particular concept; this class of functions will be later analyzed, by introducing and explaining some characterization theorems.

Theorem 3. The function $f:(X, \mathcal{F}m_X) \to (Y, t)$ is F_m-quasi-irresolute in the fuzzy point x_α in X if and only if for any F-semi-open set v in X, with $f(x_\alpha)qv$ we have $x_\alpha q\ F_m\text{- Int}(f^{-1}(\overline{v}))$.

Proof

Necessity. We assume that f is F_m-quasi-irresolute, therefore the conditions of Definition 5 are satisfied. This implies $x_\alpha\ q\delta$, $\delta \leq f^{-1}(\overline{v})$ (because $f^{-1}(f(\delta)) = \delta \leq f^{-1}(\overline{v})$ and therefore $x_\alpha\ q\ F_m\text{- Int}(f^{-1}(\overline{v}))$.

Sufficiency. Consider v a F-semi-open set in(Y, t) with $f(x_\alpha)qv$. As by hypothesis $x_\alpha q\ F_m\text{ - Int}(f^{-1}(\overline{v}))$, there exists $\delta \in \mathcal{F}m_X$ so that $x_\alpha\ q\delta$ and therefore $\delta \leq f^{-1}(\overline{v})$. This implies $f(\delta) \leq f(f^{-1}(\overline{v})) \leq \overline{v}$, which proves that f is F_m-quasi-irresolute in the fuzzy point x_α.

Theorem 4. Consider the previous spaces and the function $f:(X, \mathcal{F}m_X) \to (Y, t)$. Then the following properties are equivalent:

(1) f is F_m-quasi-irresolute;
(2) $f^{-1}(v) \leq F_m\text{- Int}(f^{-1}(\overline{v}))$, $(\forall)\, v \in \mathcal{F}(Y)$, F-semi-open in$(Y, t)$;
(3) $F_m\text{-}Cl\,(f^{-1}(\text{Int}\,\sigma)) \leq f^{-1}(\sigma)$, $(\forall)\,\sigma \in \mathcal{F}(Y)$, F-semi- closed in (Y, t);
(4) $F_m\text{-}Cl\,(f^{-1}(\text{Int}\,(\text{Fd}\,\overline{\mu}))) \leq f^{-1}(\text{Fd}\,\overline{v})$, $(\forall)\ \mu \in \mathcal{F}(Y)$;
(5) $f^{-1}(\text{Fd}\,\mathring{\mu}) \leq F_m\text{-Int}\,(\overline{Fd\,\mathring{\mu}}), (\forall)\,\mu \in \mathcal{F}(Y)$.

Proof

(1) $\Rightarrow$ (2). If $v \in \mathcal{F}(Y)$ and x_α is a fuzzy point in X with $x_\alpha q\ f^{-1}(v)$, then $f(x_\alpha)qv$, where $f(x_\alpha)$ is a fuzzy point in Y. We assume that v, F-semi-open in (Y, t) and f, F_m-quasi-irresolute in x_α. According to Theorem 3, $x_\alpha q\ F_m\text{- Int}(f^{-1}(\overline{v}))$ and therefore $f^{-1}(v) \leq F_m\text{- Int}(f^{-1}(\overline{v}))$.

(2) $\Rightarrow$ (3). If $\sigma \in \mathcal{F}(Y)$ and it is F-semi-closed in (Y, t), then σ^c is F-semi-open in (Y, t) and according to a property of f^{-1} (see [13]), according to (2) and to Lemma 1, it follows that

$(f^{-1}(\sigma))^c = f^{-1}(\sigma^c) \leq F_m\text{- Int}(f^{-1}(\overline{\sigma}^{\,c})) = F_m\text{- Int}(f^{-1}(\mathring{\sigma})^c) = F_m\text{- }(\text{Int}(f^{-1}(\mathring{\sigma})))^c = (F_m\text{- }\overline{(f^{-1}(\mathring{\sigma}))})^c$.
This implies that $F_m\text{-}Cl\,(f^{-1}(\text{Int}\,\sigma)) \leq f^{-1}(\sigma)$.

(3) $\Rightarrow$ (4). If $\mu \in \mathcal{F}(Y)$, then $\text{Fd}\,\overline{\mu}$ is F-semi- closed in (Y,t) and according to (3) there follows $F_m\text{-}Cl(f^{-1}(\text{Int}(\text{Fd}\,\overline{\mu}))) \leq f^{-1}(\text{Fd}\,\overline{\mu})$.

(4) $\Rightarrow$ (5). If $\mu \in \mathcal{F}(Y)$, then $f^{-1}(\text{Fd}\mathring{\mu}) = (f^{-1}(\text{Fd}\,\overline{\mu}^{\,c}))^c \leq (F_m\text{-}Cl(f^{-1}(\text{Int}(\text{Fd}\,\overline{\mu}^{\,c}))))^c = F_m\text{-Int}(\overline{Fd\,\mathring{\mu}})$.

(5) $\Rightarrow$ (1). Let us consider x_α a fuzzy point in X and $v \in \mathcal{F}(Y)$, F-semi-open in (Y,t) with $f(x_\alpha)qv$. This implies that $x_\alpha q f^{-1}(v) = f^{-1}(\text{Fd}\mathring{v}) \leq F_m\text{-Int}(f^{-1}(\overline{Fd\,\mathring{\mu}})) \leq F_m\text{-Int}(f^{-1}(\overline{v})) \leq F_m\text{- Int}(f^{-1}(\overline{v}))$ and therefore $x_\alpha qF_m\text{-Int}(f^{-1}(\overline{v}))$. According to Theorem 3, f is F_m-quasi-irresolute in the fuzzy point x_α.

Theorem 5. Consider the function $f: (X, \mathcal{F}m_X) \to (Y, t)$, where $\mathcal{F}m_X$ is a fuzzy supra-topology on X. Then the following properties are equivalent:

(1) f is F_m-quasi-irresolute;
(2) $f^{-1}(\sigma) \in \mathcal{F}m_X$, $(\forall)\,\sigma \in \mathcal{F}(Y)$, F-regular closed set;
(3) $f^{-1}(v^c) \in \mathcal{F}m_X$, $(\forall)\, v \in \mathcal{F}(Y)$, F-regular open set;

(4) $f^{-1}(\mu) \in \mathcal{F}m_X$, $(\forall)$ $\mu \in \mathcal{F}(Y)$, Fθ-semi-open set;

(5) $f^{-1}(\delta^c) \in \mathcal{F}m_X$, $(\forall)$ $\delta \in \mathcal{F}(Y)$, Fθ-semi-closed set.

Proof

(1) $\Rightarrow$ (2). If $\sigma \in \mathcal{F}(Y)$ is a F-regular closed set, then σ is a F-semi-open set (see [2]) and according to Theorem 4 (2), $f^{-1}(\sigma) \leq F_m\text{-Int}(f^{-1}(\overline{\sigma}))= F_m\text{-Int}(f^{-1}(\sigma))$. According to Lemma 1, $f^{-1}(\sigma) = F_m - \text{Int}(f^{-1}(\sigma))$ and according to Lemma 3, $f^{-1}(\sigma) \in \mathcal{F}m_X$.

(2) $\Rightarrow$ (3). This implication is obvious.

(3) $\Rightarrow$ (1). Any Fθ-semi-open set is an union of F-regular-closed sets and that $\mathcal{F}m_X$ is a supra-topology.

(4) $\Rightarrow$ (5). This implication is obvious.

(5) $\Rightarrow$ (1) Let us consider x_α a fuzzy point in X and $\nu \in \mathcal{F}(Y)$ a F- semi-open set with $f(x_\alpha)q\nu$. Since $\overline{\nu}$ is F-regular-closed, it is Fθ-semi-open. Then the set $\delta = f^{-1}(\overline{\nu})$ is F_m- open (according to (4)) with $x_\alpha q\delta$ and therefore $f(\delta) \leq (\overline{\nu})$, which proves that f is a F_m-quasi-irresolute function.

The concept of F_m-irresolute function is introduced by:

Definition 6. The function f: $(X, \mathcal{F}m_X) \to (Y, t)$ is called F_m-irresolute in the fuzzy point x_α in X if for any set $\nu \in \mathcal{F}(Y)$, F-semi-open in (Y, t) with $f(x_\alpha)q\nu$ there exists $\delta \in \mathcal{F}m_X$ with $x_\alpha q\delta$ so that $f(\delta) \leq \nu)$. The function f is F_m-irresolute on X if it has this property in all the fuzzy points in X.

The following characterization theorem holds:

Theorem 6. The function f:$(X, \mathcal{F}m_X) \to (Y, t)$ is F_m-irresolute if and only if for any set ν, F-semi-open in (Y, t), we have $f^{-1}(\nu)= F_m\text{-Int}(f^{-1}(\nu))$.

Proof

Necessity. Consider f, a F_m-irresolute function, therefore the conditions of Definition 6 are satisfied. This implies that $\delta \leq f^{-1}(\nu)$ and therefore $x_\alpha \in \delta \leq f^{-1}(\nu)$, where, from $x_\alpha \in F_m\text{-Int}(f^{-1}(\nu))$, we conclude that $f^{-1}(\nu) \leq F_m\text{-Int}(f^{-1}(\nu))$. According to Lemma 1 (5), $F_m\text{-Int}(f^{-1}(\nu)) \leq f^{-1}(\nu)$. We obtain $f^{-1}(\nu)= F_m\text{-Int}(f^{-1}(\nu))$.

Sufficiency. Consider ν a F-semi-open set in (Y, t), so that $f^{-1}(\nu) = F_m\text{-Int}(f^{-1}(\nu))$ and x_α a fuzzy point in X so that $x_\alpha q f^{-1}(\nu)$. Then there exists $\delta \in \mathcal{F}m_X$ with $x_\alpha q\delta$ and therefore $\delta \leq f^{-1}(\nu)$. This implies that $f(\delta) \leq \nu$, which proves that f is a F_m-irresolute function.

Remark 5. Obviously, any F_m-irresolute function is also F_m-quasi-irresolute, but generally the reciprocal of this proposition is not true. In order to obtain a true reciprocal, a supplementary condition needs to be introduced - the F-interiority condition, defined by:

Definition 7. We say that the function f: $(X, \mathcal{F}m_X) \to (Y, t)$ satisfies the F-interiority condition if $F_m\text{-Int}(f^{-1}(\overline{\nu})) \leq f^{-1}(\nu)$ for any set $\nu \in \mathcal{F}(Y)$, F-semi-open in (Y, t).

Theorem 7. If the function f: $(X, \mathcal{F}m_X) \to (Y, t)$ is F_m-quasi-irresolute and it satisfies the F-interiority condition, then f is F_m-irresolute.

Proof. Consider f F_m-quasi-irresolute. Then, if $\nu \in \mathcal{F}(Y)$ is a F-semi-open set in (Y, t), according to Theorem 4 (2), we obtain the relation $f^{-1}(\nu) \leq F_m\text{-Int}(f^{-1}(\overline{\nu}))$. According to the F-interiority condition of f and according to Lemma 1, we obtain

$$f^{-1}(\nu) \leq F_m\text{-Int}(f^{-1}(\overline{\nu})) = F_m\text{-Int}(F_m\text{-Int}\,(f^{-1}(\nu))) \leq F_m\text{-Int}(f^{-1}(\nu)),$$

which implies $f^{-1}(\nu)= F_m\text{-Int}(f^{-1}(\nu))$; this proves, by applying Theorem 6, that the function f is F_m-irresolute.

According to the above mentioned propositions, if the F-interiority condition is satisfied, the concepts of F_m-irresolute function and F_m-quasi-irresolute function are equivalent. The concepts of F_m-contra-continuous function and F_m-almost-contra-continuous function have been introduced in [5] and [6].

Definition 8.The function f: $(X, \mathcal{F}m_X) \rightarrow (Y, t)$ is called

(a) F_m-contra-continuous if $f^{-1}(v)= F_m\text{-}Cl\,(f^{-1}(v))$, $(\forall)\, v \in t$;

(b) F_m-almost-contra-continuous if $f^{-1}(v)= F_m\text{-}Cl\,(f^{-1}(v))$ for any set $v \in \mathcal{F}(Y)$, F-regular-open.

In [6] we have established the equivalence between the concepts of F_m-almost-contra-continuous function and F_m-quasi-irresolute function (see Theorem 5 in [6]).

In conclusion, if the F-interiority condition is satisfied, the concepts of F_m-irresolute function, F_m-quasi-irresolute function and F_m-almost-contra-continuous function are equivalent.

References

1. Akdag, M. - Almost Forms of Some Fuzzy Continuities, *Transaction of Academy of Sciences of Azerbadjan, Series of Physical and Mathematical Sciences*, No.2, 3, 2000, pp.70-76
2. Azad, K.K. - On Fuzzy Semicontinuity, Fuzzy Almost Continuity and Fuzzy Weakly Continuity, *Journal of Mathematical Analysis and Applications,* No. 82, 1981, pp. 14-32
3. Brescan, M. - Applications Floues Irrésolutes. Applications Floues Quasi- Irrésolutes, *Studii şi Cercetări Ştiinţifice, Seria Matematică,* Universitatea din Bacău , No. 8, 1968, pp. 9-18
4. Brescan, M. - Structures Floues Minimaux, *Petroleum-Gas University of Ploieşti Bulletin, Mathematics-Informatics-Physics Series,* vol. LIII, No.1, 2001, pp.115-122
5. Brescan, M. - Contre-Continuite dans les Structures Floues Minimaux, *Petroleum-Gas University of Ploieşti Bulletin, Mathematics-Informatics-Physics Series*, No.2, 2008, pp.17-26
6. Brescan, M.- Popa, C., Presque Contre-Continuite dans les Structures Floues Minimaux, *Scientific Studies and Research, Mathematics Series*, University of Bacău , No.19, 2009, pp. 33-48
7. Chang, C.L. - Fuzzy Topological Spaces, *Journal of Mathematical Analysis and Applications,* No.43, 1973, pp. 734-742
8. Min, W.K., Park, C.K. - On Fuzzy M-sets and Fuzzy M-Continuity, *International Journal of Fuzzy Logic and Intelligent Systems,* March 2005, vol.5, No.1, pp .59-63
9. Ming, P.P., Ming, L.Y. - Fuzzy Topology I, *Journal of Mathematical Analysis and Applications*, 77, 1980, pp. 20-37
10. Noiri, T., Popa, V. - On M-continuous functions, *Analele Universităţii "Dunărea de Jos",* Galaţi, Fascicola II, XVIII (2000), pp. 31-41
11. Noiri, T., Popa, V. - On the definitions of some generalized forms of continuity under minimal conditions, *Memoirs Faculty of Sciences Kochi University Series A, Mathematics*, 22, 200l
12. Noiri, T., Popa, V. - On *m*-Quasi-Irresolute Functions, *Mathematica Moravica*, 2005, vol. 9, pp. 25-41
13. Tuna, H., Yalvaç, S. - Fuzzy Sets and Functions of Fuzzy Spaces, *Journal of Mathematical Analysis and Applications*, 126, 1987, pp. 409-423

Funcţii quasi-irresolute în structuri fuzzy minimale

Rezumat

Scopul acestei lucrări este de a generaliza pentru o structură fuzzy minimală conceptul de funcţie fuzzy m-quasi-irresolută introdus în Topologia generală de Takashi Noiri şi Valeriu Popa. Se dau câteva teoreme de caracterizare importante şi se pun în evidenţă echivalenţe între unele noţiuni.